

MAT511 homework, due Nov 12, 2003

- (1) Suppose that  $A$  is a finite set with  $m$  elements, and  $B$  is a finite set with  $n$  elements.
- (a) Find the total number of functions from  $A$  to  $B$  if
- $m = n$
  - $m > n$
  - $m < n$
- (b) Find the number of one-to-one functions from  $A$  to  $B$  if
- $m = n$
  - $m > n$
  - $m < n$
- (2) Give an example of functions  $f : A \rightarrow B$  and  $g : B \rightarrow C$ , (be sure to specify domains and ranges) for which
- (a)  $g$  is onto  $C$ , but  $g \circ f$  is not onto  $C$ .
- (b)  $g \circ f$  is onto  $C$ , but  $f$  is not onto  $B$ .
- (c)  $g$  is one-to-one, but  $g \circ f$  is not one-to-one.
- (d)  $g \circ f$  is one-to-one but  $g$  is not one-to-one.
- (3) Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be given by  $f(x) = x^2 + 1$ . Find the following (remember that in this context  $[a, b]$  is the set  $\{x \in \mathbb{R} \mid a \leq x \leq b\}$ ).
- (a)  $f([1, 2])$
- (b)  $f([-1, 2])$
- (c)  $f^{-1}([5, 10])$
- (d)  $f^{-1}([-1, 5] \cup [17, 26])$
- (4) Let  $f : A \rightarrow B$ , and  $D \subseteq A$ ,  $E \subseteq B$ . Prove that  $D \subseteq f(f^{-1}(D))$ . Also, Give an example where  $D \neq f(f^{-1}(D))$ .