MAT511 homework, due Nov 5, 2003

(1) Give a relation R from $A = \{5, 6, 7\}$ to $B = \{3, 4, 5\}$ such that

(a) R is not a function.

(b) R is a function from A to B, with the image of R equal to B.

(c) R is a function from A to B, with the image of R not equal to B.

(d) R is a function from A to B which is not one-to-one.

(2) Explain why the functions

$$f(x) = \frac{9 - x^2}{x + 3} \qquad \text{and} \qquad g(x) = 3 - x$$

are not equal.

(3) A **metric** on a set X is a function $d : X \times X \to \mathbb{R}$ so that for all x, y, and z in X, the following properties are satisfied:

- $d(x, y) \geq 0$
- $d(x, y) = 0$ if and only if $x = y$.
- $d(x, y) = d(y, x)$
- $d(x, y) + d(y, z) \geq d(x, z)$

Prove that each of the following is a metric for the indicated set.

the Euclidean metric: $X = \mathbb{R}$, $d(x, y) = \sqrt{(x - y)^2}$

the Manhattan metric: $X = \mathbb{R}^2$, $d\big((x, y), (z, w)\big) = |x - z| + |y - w|$

the discrete metric: X is any set, $d(x, y) = 0$ whenever $x = y$, and $d(x, y) = 1$ if $x \neq y$.

(4) For each of the following, decide whether they are one-to-one and whether they are onto. Prove your answers.

(a) $f : \mathbb{N} \to \mathbb{N}$, $f(x) = 2x + 1$

(b) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = 2x + 1$

(c) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = 2^x$

(d) $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, $f(x, y) = x - y$

(e) $f : (1, \infty) \to (1, \infty)$, $f(x) = \dfrac{x}{x - 1}$

(5) Prove that if a real-valued function f is strictly increasing, then f is one-to-one. Also, give an example of a real-valued function g which is strictly increasing, but is not onto.