MAT511 homework, due Oct 29, 2003

(1) Define the relation $\trianglelefteq$ on $\mathbb{R} \times \mathbb{R}$ by $(a, b) \trianglelefteq (c, d)$ if and only if $a \leq c$ and $b \leq d$. Prove that this relation is a partial ordering on $\mathbb{R} \times \mathbb{R}$.

(2) Let A be a partially ordered set, which we call the "alphabet". A "string" (or a "word") is a finite sequence of elements of A (written strung all together). Let $\mathcal{W}_A$ be the set of all strings made from elements of A. For example, if $A = \{a, b, c\}$, then a, $abba$, $baccababa$, and $\emptyset$ are all elements of $\mathcal{W}_A$, where $\emptyset$ denotes the empty string which is of length zero.

If σ and τ are two strings in $\mathcal{W}_A$, then let $\sigma \smile \tau$ be the concatenation of σ and τ. For example, if σ is the string $floo$ and τ is $baru$, then $\sigma \smile \tau$ is $floobaru$. Note that for any string σ, $\sigma \smile \emptyset = \sigma$.

Define the relation $\ll$ on $\mathcal{W}_A$ by $\sigma \ll \tau$ if and only if there is a string $\nu \in \mathcal{W}_A$ so that $\tau = \sigma \smile \nu$.

Prove that $\ll$ is a partial order on $\mathcal{W}_A$.

(3) Let R be the rectangle in the cartesian plane given by

$$R = \{(x, y) \mid 0 \leq x \leq 3, 0 \leq y \leq 1\}$$

Let $\mathcal{H}$ be the set of all rectangles whose sides have positive length, are parallel to the sides of R, and are contained in R. $\mathcal{H}$ is partially ordered by set inclusion.

(a) Does every subset of $\mathcal{H}$ have an upper bound? A least upper bound? (justify your answers).

(b) Does every subset of $\mathcal{H}$ have a largest element?

(c) Does every subset of $\mathcal{H}$ have a lower bound? A greatest lower bound?

(d) Does every subset of $\mathcal{H}$ have an smallest element?