MAT511 homework, due Oct 22, 2003

(1) Let A and B be nonempty sets. Prove that $A \times B = B \times A$ if and only if $A = B$. What if one of A or B is empty?

(2) For each of the relations below, indicate whether it is reflexive, symmetric, or transitive. Justify your answer.
 (a) $\leq$ on the set $\mathbb{N}$.
 (b) $\perp = \{(l, m) \mid l$ and m are lines, with l perpendicular to m $\}$.
 (c) $\sim$ on $\mathbb{R} \times \mathbb{R}$, where $(x, y) \sim (z, w)$ if $x + z \leq y + w$.
 (d) $\smile$ on $\mathbb{R} \times \mathbb{R}$, where $(x, y) \smile (z, w)$ if $x + y \leq z + w$.
 (e) $\square$ on $\mathbb{R} \times \mathbb{R}$, where $(x, y) \square (z, w)$ if $x + z = y + w$.

(3) Prove that if R is a symmetric, transitive relation on a set A, and the domain of R is A, then R is reflexive on A.

(4) Consider the relations $\sim$ and $\square$ on $\mathbb{N}$ defined by $x \sim y$ iff $x + y$ is even, and $x \square y$ iff $x + y$ is a multiple of 3. Prove that $\sim$ is an equivalence relation, and that $\square$ is not.

(5) For each $a \in \mathbb{R}$, let $P_a = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y = a - x^2\}$.
 (a) Sketch the graph of P_{-2}, P_0, and P_1.
 (b) Prove that $\{P_a \mid a \in \mathbb{R}\}$ forms a partition of $\mathbb{R} \times \mathbb{R}$.
 (c) Describe the equivalence relation associated with this partition.