(1) Use induction to prove that, for all natural numbers $n$,

$$
1+4+7+\ldots+(3 n-2)=\frac{n(3 n-1)}{2}
$$

(2) Prove that for any natural number $n, n^{3}+5 n+6$ is divisible by 3 .
(3) If a set $A$ has $n$ elements, prove that its power set $\mathcal{P}(A)$ has $2^{n}$ elements.
(4) Let $P_{1}, P_{2}, \ldots P_{n}$ be $n$ points in a plane, no three of which are collinear. Prove (by induction) that the number of line segments joining all pairs of points is $\left(n^{2}-n\right) / 2$.


