MAT511 homework, Sept. 17, 2003

Let A, B, C be sets.

(1) Prove that $A \subseteq B$ if and only if $A - B = \emptyset$.

(2) Prove that $C \subseteq (A \cap B)$ if and only if $C \subseteq A$ and $C \subseteq B$.

(3) Prove that $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$. You may use the results above. (note that the earlier version of this problem had a typo. Sorry)

(4) Prove that $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.

(5) Give an example where $\mathcal{P}(A) \cup \mathcal{P}(B) \neq \mathcal{P}(A \cup B)$. What conditions are necessary on A and B to ensure that $\mathcal{P}(A) \cup \mathcal{P}(B) = \mathcal{P}(A \cup B)$?

(6) Show that there are no sets A and B for which $\mathcal{P}(A - B) = \mathcal{P}(A) - \mathcal{P}(B)$.

(7) Let $\mathcal{A}$ be the family of all sets of integers containing 10. What are the sets $\bigcup_{A \in \mathcal{A}} A$ and $\bigcap_{A \in \mathcal{A}} A$? Justify your answer.

(8) Let $A_n = \left[\frac{1}{n}, 2 + \frac{1}{n}\right]$. What are the sets $\bigcup_{n \in (\mathbb{N} - \{1,2\})} A_n$ and $\bigcap_{n \in (\mathbb{N} - \{1,2\})} A_n$? Justify your answer.

(9) Let $\mathcal{A}$ and $\mathcal{B}$ be two pairwise disjoint families of sets. Let $\mathcal{C} = \mathcal{A} \cap \mathcal{B}$, and $\mathcal{D} = \mathcal{A} \cup \mathcal{B}$.
   (a) Prove that $\mathcal{C}$ is a pairwise disjoint family of sets.
   (b) Give an example where $\mathcal{D}$ is not a pairwise disjoint family of sets.
   (c) Prove that if the sets $\bigcup_{A \in \mathcal{A}} A$ and $\bigcup_{B \in \mathcal{B}} B$ are disjoint, then $\mathcal{D}$ is a pairwise disjoint family.