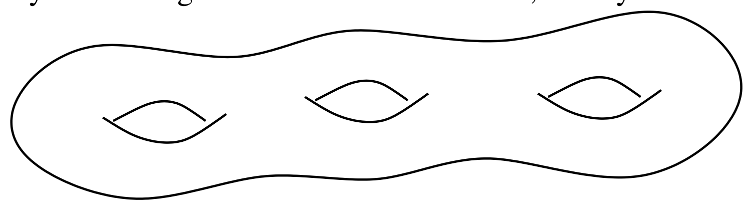
MAT364, More homeworks

1. Give an explicit calculation of the Euler characteristic of the two dimensional manifold of genus three. Do this in two ways, both by calculating the index of a vector field, and by constructing a triangulation.



2. Show that given a triangulation of a two-dimensional, compact manifold M without boundary, a vector field can be explicitly constructed with a zero at the center of each face, at the center of each edge, and on each vertex. Use this vector field to show explicitly that

$$\chi(M) = \#F - \#E + \#V = \sum_{zeros\ z_j} i(z_j)$$

(This method also works if M has boundary, but you have to be a little more careful, so don't worry about that).

3. Suppose $f : X \to \mathbb{S}^{n-1}$ and $g : X \to \mathbb{S}^{n-1}$ are two smooth maps from a manifold X to the sphere

$$\mathbb{S}^{n-1} = \left\{ (x_1, x_2, \ldots, x_n) \mid x_1^2 + x_2^2 + \ldots + x_n^2 = 1 \right\},$$

amd that $\|f(x) - g(x)\| < 2$ for all points $x \in X$. Show that f and g are homotopic.

4. Let M be a non-orientable 2-dimensional manifold. Show that M must contain a Möbius strip.