1. Show that \( A = \{(x, y) \mid 1 < x^2 + y^2 < 2\} \) and \( B = \mathbb{R}^2 \setminus \{(0, 0)\} \) are diffeomorphic.

More specifically, construct an explicit map \( f : B \to A \) and show that

- \( f \) is a continuous bijection
- \( Df \) is nonsingular everywhere (calculate \( Df \) and show that the determinant is nonzero for all \( (x, y) \in B \)).
- \( Df^{-1} \) is nonsingular (explicitly find \( f^{-1} : A \to B \) and calculate \( Df^{-1} \), then show the determinant is nonzero).

2. Give an explicit example (that is, write down a parameterization) of a function \( g : \mathbb{R}^2 \to \mathbb{R}^3 \) for which

- \( g \) is injective
- \( Dg(0, 0) \) has rank 0
- the rank of \( Dg(x, y) \) is 2 at all \( (x, y) \neq (0, 0) \).

3. Let \( M \) be the image of \( \mathbb{R}^2 \) under the map \( h : \mathbb{R}^2 \to \mathbb{R}^3 \) given by \( h(x, y) = (x^3, x^2, y) \).

   (a) Does \( h^{-1} : M \to \mathbb{R}^2 \) exist?
   (b) What is the tangent space to \( M \) at \( (0, 0, 0) \)?
   (c) What is the tangent space to \( M \) at \( (1, 1, 1) \)?
   (d) Is \( M \) a smooth manifold?