MAT364, Homework 2

due wednesday 9/21

1. Consider the standard roman alphabet of uppercase letters in a sans-serif font

   ABCDEFGHIJKLMNOPQRSTUVWXYZ

   Classify the letters into sets which are homeomorphic to one another, if you consider the letters as being made from line segments.

   Does your answer change if the lines which make up the letters are viewed as having width, albeit small? If so, how?

   (You don't need to give a formal proof of either of these, just an informal justification.)

2. Show that homeomorphism is an equivalence relation.

3. Each of the following maps is discontinuous. In each case, find an open set in the image whose preimage is not open in the domain.

   (a) $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = [x]$ (the integer part of x).

   (b) $g : \mathbb{R}^2 \to \mathbb{R}$ where $g(x,y) = \begin{cases} 0 & \text{if } x \geq 0 \\ 1 & \text{if } x < 0 \end{cases}$

   (c) $h : \mathbb{R} \to \mathbb{R}^3$ where $h(t) = \begin{cases} (\cos(t), \sin(t), 1) & \text{if } t \geq 0 \\ (\cos(t), \sin(t), -1) & \text{if } t < 0 \end{cases}$

4. Let $f : \mathbb{R}^n \to \mathbb{R}^n$ be a map which decreases all distances, that is $d(f(x), f(y)) < d(x, y)$ for all $x, y \in \mathbb{R}^n$. Prove that f is continuous.