1. Recall that for a set $A$ in $\mathbb{R}^n$, its closure is the union of its interior points and those in its frontier, that is

$$\text{Cl}(A) = \text{Int}(A) \cup \text{Fr}(A).$$

Prove for any such $A$, $\text{Cl}(A)$ is a closed set.

2. Suppose $x$ and $y$ are points in $\mathbb{R}^n$. The closed line segment between $x$ and $y$ is defined as

$$[x, y] = \{tx + (1-t)y \mid 0 \leq t \leq 1\}.$$

A subset $A \subseteq \mathbb{R}^n$ is called convex if for every $x$ and $y$ in $A$, every point of the segment $[x, y]$ is also contained in $A$.

Prove that if $x_0$ is any point in $\mathbb{R}^n$, the open disk $D_r(x_0)$ and the closed ball $B_r(x_0)$ are both convex sets.

3. Give an example of a countably infinite family of closed sets $A_1, A_2, A_3, \ldots$ such that $\bigcup_{i=1}^{\infty} A_i$ is not closed.

4. True or False: “If $A$ and $A \cup B$ are open, then $B$ must be open. If true, give a proof. If false, give a counterexample.

5. Consider the real line $\mathbb{R}$ as the $x$-axis in $\mathbb{R}^2$. If $B$ is a closed subset of $\mathbb{R}$, prove that it is also closed when viewed as a subset of $\mathbb{R}^2$.

Is the same property true for open sets? That is, if $A$ is an open subset of $\mathbb{R}$, is $A$ also open when viewed as a subset of $\mathbb{R}^2$? Prove or give a counterexample.