37. (*expires 5/4*) Find all the fixed points of the system

$$\begin{cases} \dot{x} &= x^2 + y, \\ \dot{y} &= x(y^2 - 1), \end{cases}$$

where a "fixed point" is a solution for which **both** $x(t)$ **and** $y(t)$ are constant. For each of these solutions you find, describe the behavior of the solutions that have initial conditions nearby. You can use `Maple` to figure out what happens for nearby points, or you can use more mathematical methods.

38. (*expires 5/4*) Consider the differential equation $\dot{\mathbf{z}}(t) = \mathbf{F}(\mathbf{z}(t))$, where the vector $\mathbf{z}(t) = (x(t), y(t))$ and the field $\mathbf{F}(x, y) = (-y, x - y)$. Plot a few solutions. What happens to them when $t \to +\infty$? Give a "`Maple`-proof" that this is a general fact for *every* solution. [*A "`Maple`-proof" is an argument that is rigorous once we accept `Maple` results as incontrovertibly true.*]

39. (*expires 5/4*) For the equation $\dot{\mathbf{z}} = \mathbf{F}(\mathbf{z})$, $\mathbf{z} = (x, y)$, with the vector field

$$\mathbf{F}(x, y) = \left\langle -x(x^4 + y^4) - y \,,\, x - y(x^4 + y^4) \right\rangle,$$

prove that the origin is an attractor in the future, i.e., every solution verifies

$$\lim_{t \to +\infty} \mathbf{z}(t) = 0.$$

Note that in this case, while you can (and probably should) use `Maple` to get an understanding and do calculations, you should format your answer as a regular mathematical proof.

40. (*expires 5/4*) We will study the Lotke-Volterra predator-prey equations: In a very simple ecosystem, at the time t (which is expressed, say, in years), there is a population of $f(t)$ foxes and $r(t)$ rabbits. The evolution of these quantities obeys the system

$$\begin{cases} \dot{f}(t) &= G_f\, f(t) + E\, f(t)\, r(t), \\ \dot{r}(t) &= G_r\, r(t) - E\, f(t)\, r(t); \end{cases}$$

where G_f and G_r are the growth rates for the foxes and the rabbits, respectively, in the absence of each other. E is the probability of a fatal encounter between a fox and a rabbit (normalized per number of foxes and rabbits).

First, write some words to explain why these equations make sense. Then, fix $G_f = 0.4$, $G_r = 2.4$ (it's well-known that rabbits have the tendency to reproduce quickly) and $E = 0.01$. For a few initial conditions of your choice, plot the trajectories in the (f, r)-plane (say, with $0 \leq f \leq 1000$ and $0 \leq r \leq 1000$). For the same initial conditions, plot the actual solutions too (i.e, $f(t)$ against t, and $r(t)$ against t). Write some comments interpreting how the behaviour of the solutions relates to what happens to the two species.

Finally, repeat the same procedure with $G_f = -1.1$. Things change substantially. Again, what is the "physical" interpretation of this?