

26. (expires 4/19) Find a formula for all the solutions to the differential equation

$$\frac{dx}{dt}(t) = -2x(t), \quad t \in \mathbb{R}.$$

Among them, single out the one for which $x(0) = 3$. [Hint: read the help page for `dsolve`, or just do it in your head. It is that easy.]

27. (expires 4/19) Have Maple find analytic solutions to the following system of differential equations,

$$\begin{cases} y''(t) - z(t) = e^t, \\ z'(t) - y(t) = 0, \end{cases}$$

with initial conditions: $y(0) = 1$, $y'(0) = 0$, $z(0) = k$. Let us denote the solutions by $y_k(t)$, $z_k(t)$ (since they depend on the parameter k).

For k taking all integer values from -10 to 10, and $t \in [-4, 2]$, plot the functions y_k in blue, and the functions z_k in red, all on the same graph. (Yes, you will then have 42 functions plotted on the same graph.) [This is certainly a case when you don't want to retype the functions that Maple finds. You will almost certainly need to read the help page for `dsolve`. I also found `subs`, `unapply`, and `seq` useful.]

28. (expires 4/19) For the functions $y_k(t)$ and $z_k(t)$ found in the previous problem, plot the parametric curves $\varphi_k(t) = [y_k(t), z_k(t)]$ for integer values of k between -5 and 5 and $-6 < t < 4$ on the same graph. Use the `view` option of `plot` to only show what lies in the region $-10 < y < 10$, $-10 < z < 10$, and use a sequence of colors so that each solution is a different color. [HINT: you might find something like `seq(COLOR(HUE, i/11), i=0..10)` useful for the latter.]

29. (expires 4/19) Find all the fixed points of the system

$$\begin{cases} \dot{x} = x^2 + y, \\ \dot{y} = x(y^2 - 1), \end{cases}$$

where a "fixed point" is a solution for which **both** $x(t)$ **and** $y(t)$ are constant. For each of these solutions you find, describe the behavior of the solutions that have initial conditions nearby. You can use Maple to figure out what happens for nearby points, or you can use more mathematical methods.

NOTE: The fact that there are various notations for differential equations is purely intentional.