

NOTE: Each exercise is worth 10 points and can be turned in at any time before its “expiration date”. At the end of the semester, I will expect you to have turned in at least 1/2 of the exercises assigned. If you do more, I will pick your best grades. If you do less, the missing grades will be counted as zeros. Altogether, these will count the same as one project.

Many of these problems will require you to use the help system and/or read the text to figure out what commands you need to use and how to use them.

1. (expires 2/6) If $p(x)$ is a complex polynomial with real coefficients, it is well known that it can be factored into a product of linear and quadratic terms with real coefficients, or into a product of linear terms only if the coefficients are allowed to be complex.

First, use `Maple` to write $q(z) = x^5 - 3x^4 - 3x^3 + 9x^2 - 10x + 30$ as a product of *exact* linear and quadratic factors with real coefficients. By exact, I mean you should leave any non-rational factors expressed as radicals; do not approximate terms like $\sqrt{3}$ as 1.73205, etc.

Then write $q(x)$ as a product of only linear factors (which will involve complex numbers).

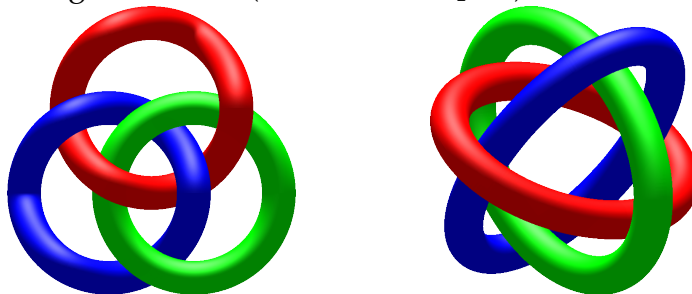
Finally, do the same for product $p(x) = x^5 - 2x^4 - 10x^3 + 20x^2 - 16x + 32$.

Hint: Note that this question asks for four different answers, two for each polynomial. While the maple command `factor` is relevant, it will need a little assistance to be able to answer all four parts. Alternatively, there are other ways to do this.

2. (expires 2/6) Draw a graph showing both $\cos(x)$ and its fifth Taylor polynomial (that is, $1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4$) for x between -4 and 4 . What degree of Taylor polynomial seems to be needed to get good agreement in this range? *Hint:* use a variation of the command `convert(taylor(cos(x), x, 5), polynom)` to make this work. Think of a suitable way to demonstrate that the approximation you have taken is “good” – what is a good definition of “good” here? Be sure to **explain** your choice of the meaning of “good”.
3. (expires 2/13) The Borromean rings are a set of three mutually interlocked rings, arranged so that if one ring is removed, the other two become unlinked.

Use `Maple` to make an image of the Borromean rings in \mathbb{R}^3 . *Hint:* I suggest using `tubeplot` from the `plots` package. The real challenge of this problem is figuring out how to parameterize the three curves.

Note that the Borromean rings can not be made from flat, round rings. Either some of the rings have to be bent a little to weave through each other, or flat rings which are elliptical can be used. See the figures below (made with `Maple`).



4. (expires 2/13) Plot the function $f(x) = 2 \sin x - x^3 - 1/5$, for $x \in [-4, 4]$. Find all the zeros of the function with an accuracy of 20 decimal digits. *Hint:* See `Digits`, `fsolve`.