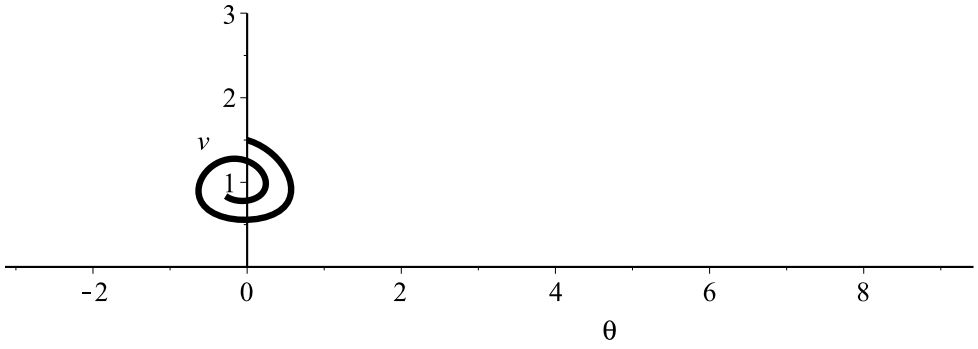
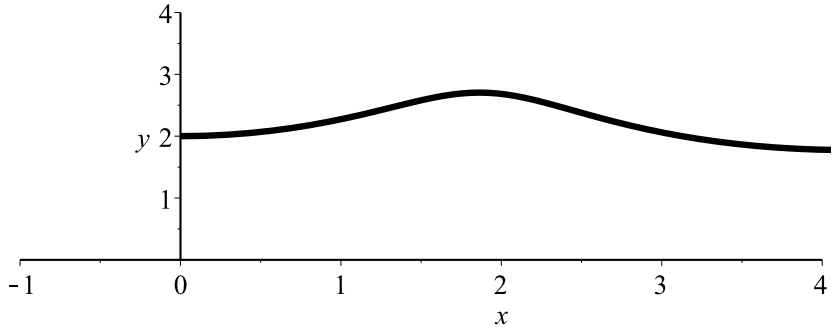
```
>
> with(plots):
  with(DEtools):
> xphug:=R-> [diff(theta(t),t) = (v(t)^2 - cos(theta(t)))/(v(t)),
              diff(v(t),t)     = -sin(theta(t))-R*v(t)^2,
              diff(x(t),t)     = v(t)*cos(theta(t)),
              diff(y(t),t)     = v(t)*sin(theta(t))];
```

$$xphug := R \rightarrow \left[\frac{d}{dt}\theta(t) = \frac{v(t)^2 - \cos(\theta(t))}{v(t)}, \frac{d}{dt}v(t) = -\sin(\theta(t)) - R\,v(t)^2, \frac{d}{dt}x(t) = v(t)\cos(\theta(t)), \frac{d}{dt}y(t) = v(t)\sin(\theta(t)) \right] \quad (1)$$

```
> DEplot(xphug(.1), [theta,v,x,y], t=0..7,
      [[theta(0)=0, v(0)=1.5, x(0)=0, y(0)=3]],
      theta=-Pi..3*Pi, v=0..3, x=-1..4, y=0..4,
      linecolor=black, stepsize=0.1, obsrange=false,
      scene=[theta,v]);
```

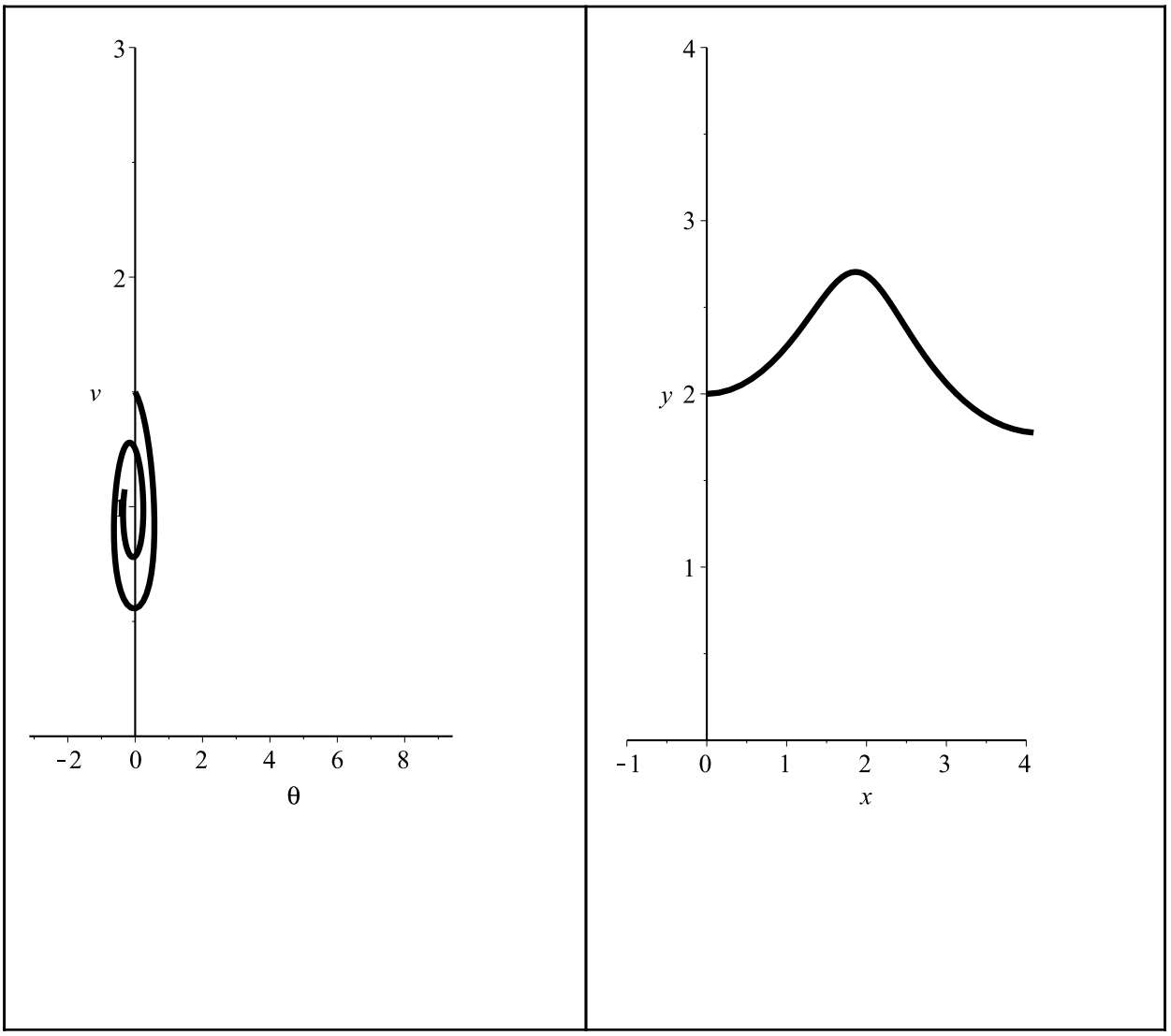



```
> DEplot(xphug(.1), [theta,v,x,y], t=0..7,
      [[theta(0)=0, v(0)=1.5, x(0)=0, y(0)=2]],
      theta=-Pi..3*Pi, v=0..3, x=-1..4, y=0..4,
      linecolor=black, stepsize=0.1, obsrange=false,
      scene=[x,y]);
```



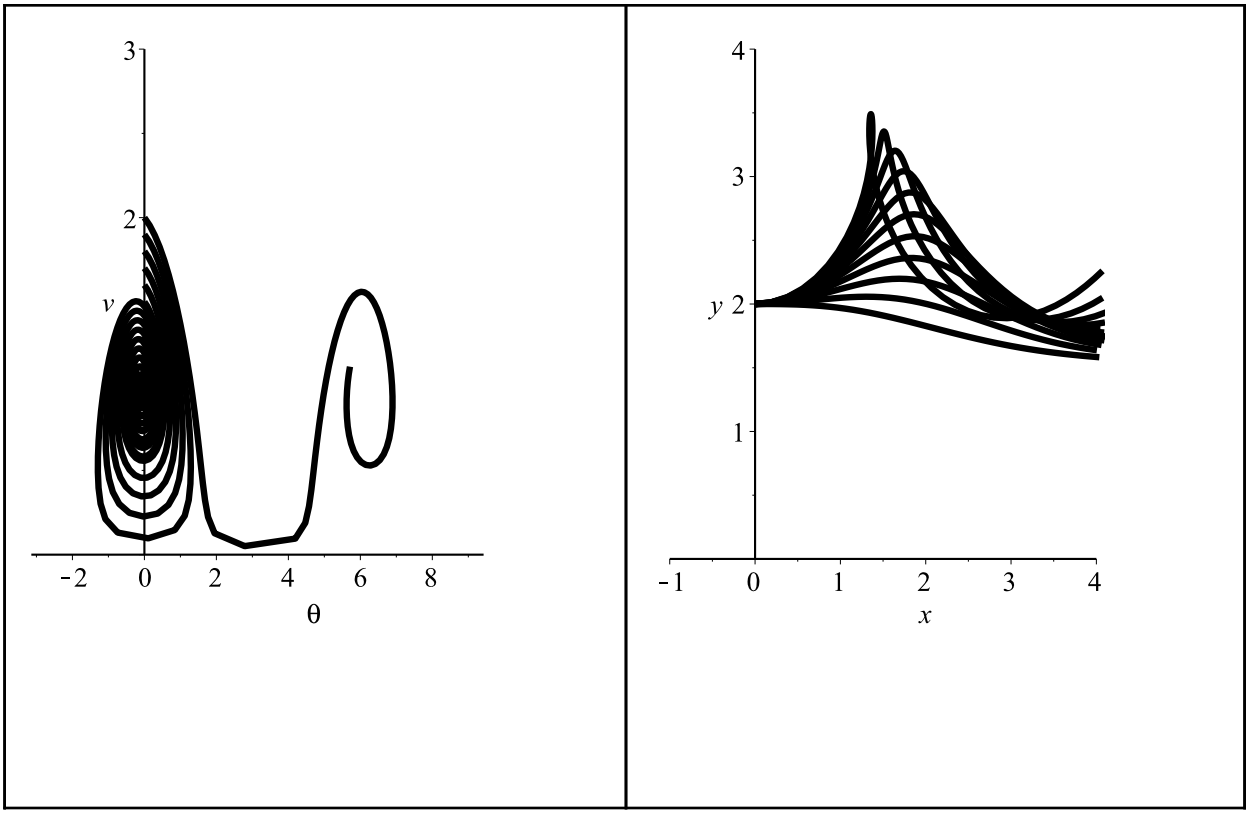

```
> display(
   Array([DEplot(xphug(.1), [theta,v,x,y], t=0..8,
      [[theta(0)=0, v(0)=1.5, x(0)=0, y(0)=2]],
      theta=-Pi..3*Pi, v=0..3, x=-1..4, y=0..4,
```

```
    linecolor=black, stepsize=0.1, obsrange=false,
     scene=[theta,v]),
  DEplot(xphug(.1), [theta,v,x,y], t=0..8,
     [[theta(0)=0, v(0)=1.5, x(0)=0, y(0)=2]],
     theta=-Pi..3*Pi, v=0..3, x=-1..4, y=0..4,
     linecolor=black, stepsize=0.1, obsrange=false,
      scene=[x,y])]));
```

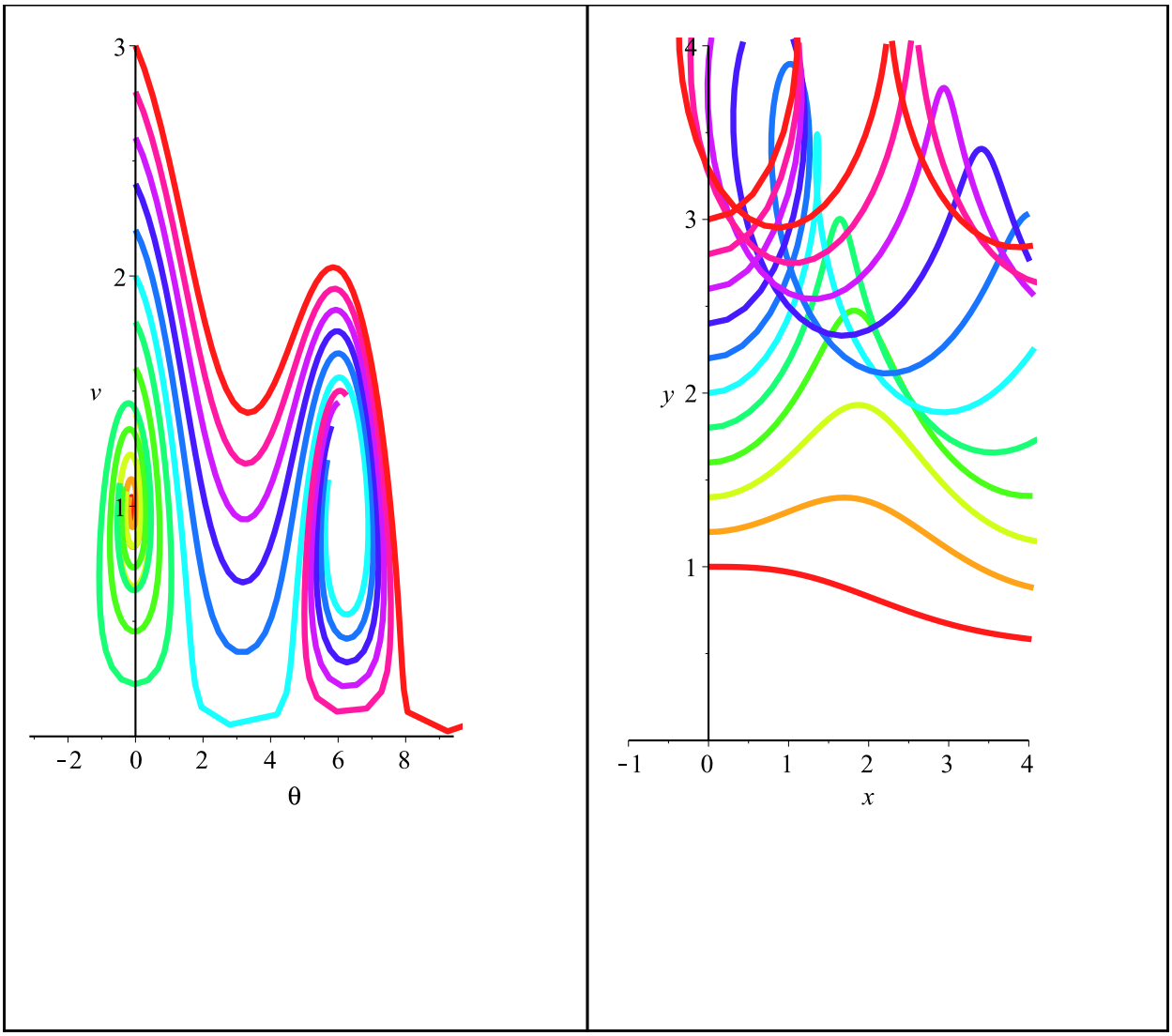



```
> Plot4 := proc(vmin, vmax, numv)
   local inits,vi;
   inits:= [seq( [theta(0)=0, v(0)=vi, x(0)=0, y(0)=2],
     vi=vmin..vmax, (vmax-vmin)/numv )];
   display(
   Array([DEplot(xphug(.1), [theta,v,x,y], t=0..8,
        inits,
     theta=-Pi..3*Pi, v=0..3, x=-1..4, y=0..4,
     linecolor=black, stepsize=0.1, obsrange=false,
      scene=[theta,v]),
   DEplot(xphug(.1), [theta,v,x,y], t=0..8,
        inits,
     theta=-Pi..3*Pi, v=0..3, x=-1..4, y=0..4,
     linecolor=black, stepsize=0.1, obsrange=false,
```

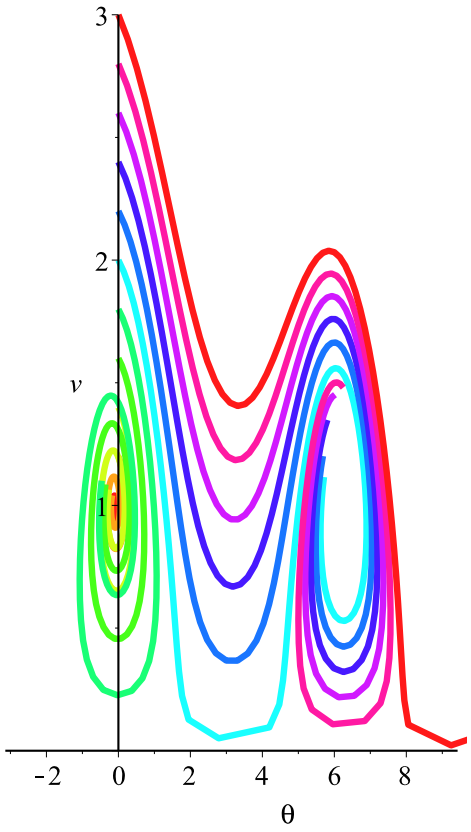
```
        scene=[x,y])]))
    end:
> Plot4(1,2,10);
```

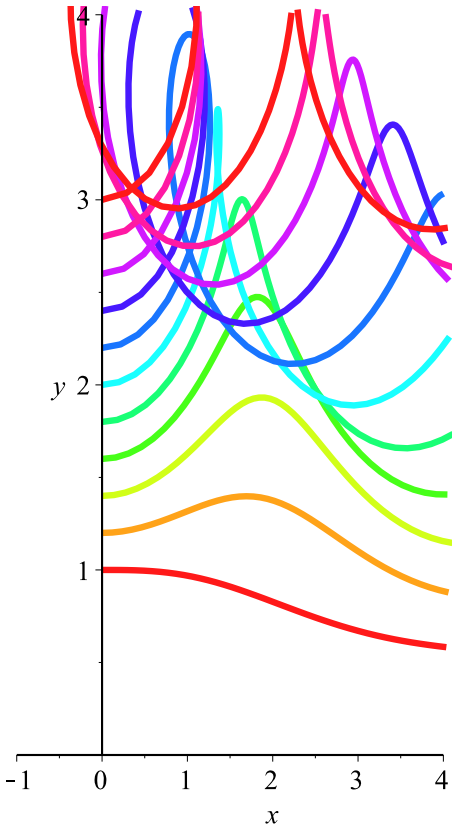



```
> Plot4 := proc(vmin, vmax, numv)
    local inits,vi,i,cols;
    inits:= [seq( [theta(0)=0, v(0)=vi, x(0)=0, y(0)=vi],
      vi=vmin..vmax, (vmax-vmin)/numv )];
    cols:= [seq(COLOR(HUE,i),i=0..1,1/numv)];
    display(
    Array([DEplot(xphug(.1), [theta,v,x,y], t=0..8,
          inits,
         theta=-Pi..3*Pi, v=0..3, x=-1..4, y=0..4,
        linecolor=cols, stepsize=0.1, obsrange=false,
         scene=[theta,v]),
     DEplot(xphug(.1), [theta,v,x,y], t=0..8,
          inits,
         theta=-Pi..3*Pi, v=0..3, x=-1..4, y=0..4,
        linecolor=cols, stepsize=0.1, obsrange=false,
         scene=[x,y])]))
    end:
> Plot4(1,3,10);
```

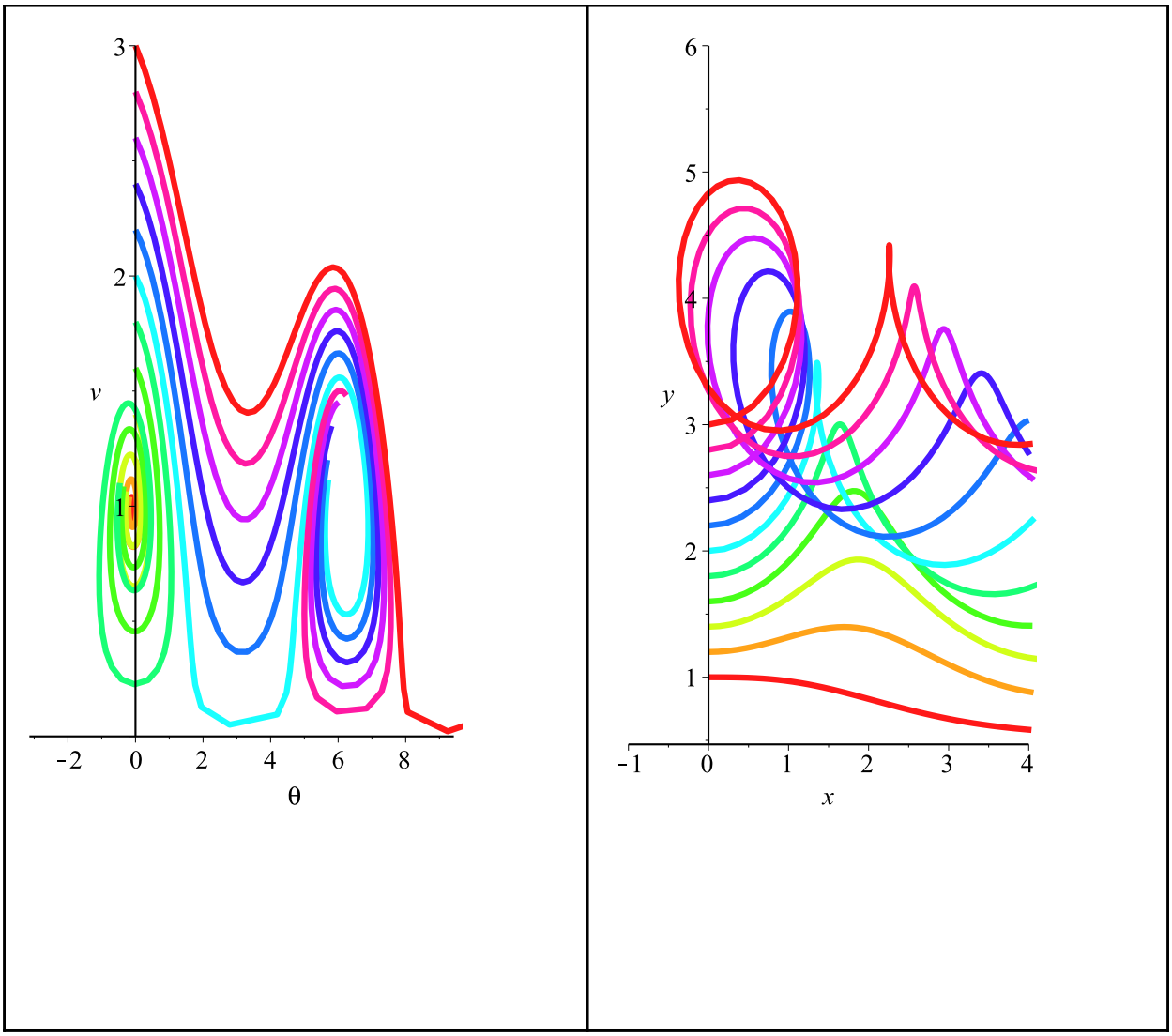



```
> Plot4 := proc(vmin, vmax, numv, {yrange:=0..4})
   local inits,vi,i,cols;
   inits:= [seq( [theta(0)=0, v(0)=vi, x(0)=0, y(0)=vi],
     vi=vmin..vmax, (vmax-vmin)/numv )];
   cols:= [seq(COLOR(HUE,i),i=0..1,1/numv)];
   display(
   Array([DEplot(xphug(.1), [theta,v,x,y], t=0..8,
         inits,
        theta=-Pi..3*Pi, v=0..3, x=-1..4, y=yrange,
       linecolor=cols, stepsize=0.1, obsrange=false,
        scene=[theta,v]),
    DEplot(xphug(.1), [theta,v,x,y], t=0..8,
         inits,
        theta=-Pi..3*Pi, v=0..3, x=-1..4, y=yrange,
       linecolor=cols, stepsize=0.1, obsrange=false,
        scene=[x,y])]))
   end:
> Plot4(1,3,10);
```

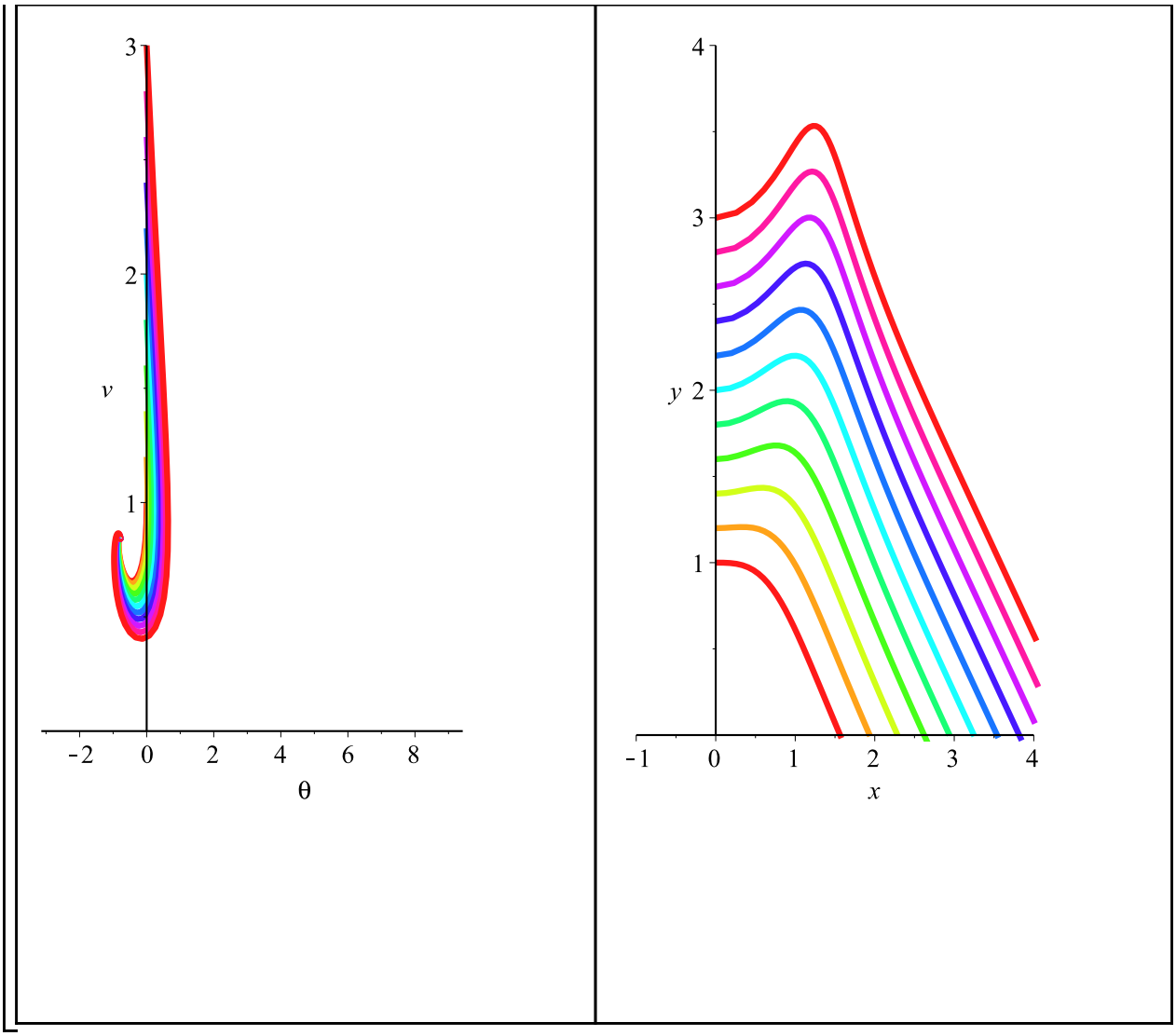




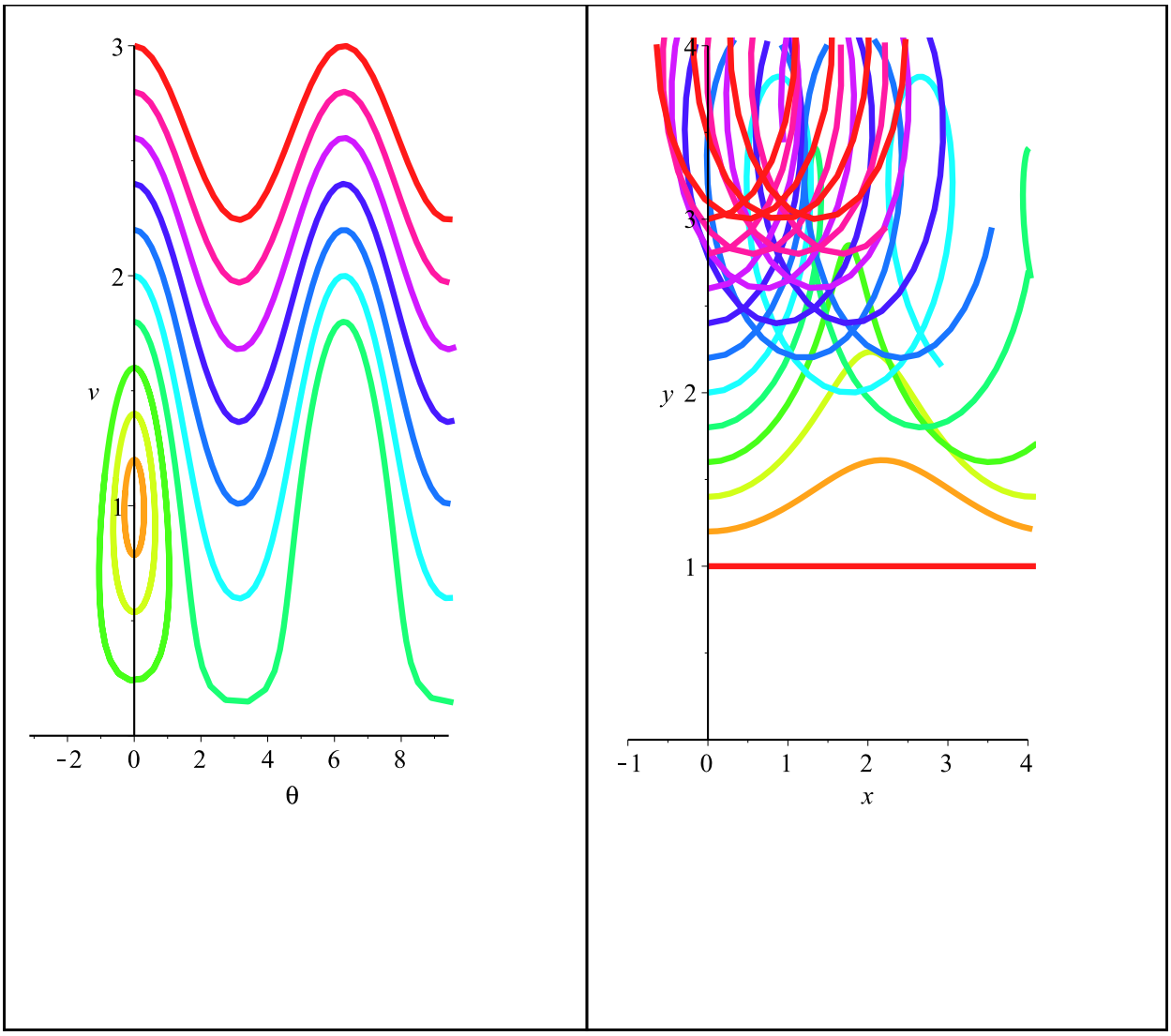

```
> Plot4(1,3,10,yrange=.5..6);
```




```
> Plot4 := proc(vmin, vmax, numv, {yrange:=0..4, R:=0})
   local inits,vi,i,cols;
   inits:= [seq( [theta(0)=0, v(0)=vi, x(0)=0, y(0)=vi],
     vi=vmin..vmax, (vmax-vmin)/numv )];
   cols:= [seq(COLOR(HUE,i),i=0..1,1/numv)];
   display(
   Array([DEplot(xphug(R), [theta,v,x,y], t=0..8,
         inits,
        theta=-Pi..3*Pi, v=0..3, x=-1..4, y=yrange,
       linecolor=cols, stepsize=0.1, obsrange=false,
        scene=[theta,v]),
    DEplot(xphug(R), [theta,v,x,y], t=0..8,
         inits,
        theta=-Pi..3*Pi, v=0..3, x=-1..4, y=yrange,
       linecolor=cols, stepsize=0.1, obsrange=false,
        scene=[x,y])]))
   end:
> Plot4(1,3,10,R=1);
```




```
> display( [seq(Plot4(1,3,10,R=r),r=0..2,.05)], insequence=true);
```




```
> xphug(R);
```

$$\left[\frac{d}{dt}\theta(t)=\frac{v(t)^2-\cos(\theta(t))}{v(t)},\ \frac{d}{dt}v(t)=-\sin(\theta(t))-R\,v(t)^2,\ \frac{d}{dt}x(t)=v(t)\cos(\theta(t)),\ \frac{d}{dt}y(t)=v(t)\sin(\theta(t))\right] \quad (2)$$

```
> solve( {v^2 - cos(theta)=0, -sin(theta)-R*v^2=0}, {v,theta});
```

$$\{\theta=\arctan(-R\,RootOf(-1+(R^2+1)\_Z^2),\ RootOf(-1+(R^2+1)\_Z^2)),\ v=RootOf(-RootOf(-1+(R^2+1)\_Z^2)+\_Z^2)\} \quad (3)$$

```
> convert(%,radical);
```

$$\left\{\theta=\arctan\left(-R\sqrt{\frac{1}{R^2+1}},\sqrt{\frac{1}{R^2+1}}\right),\ v=\left(\frac{1}{R^2+1}\right)^{1/4}\right\} \quad (4)$$