```
>
```

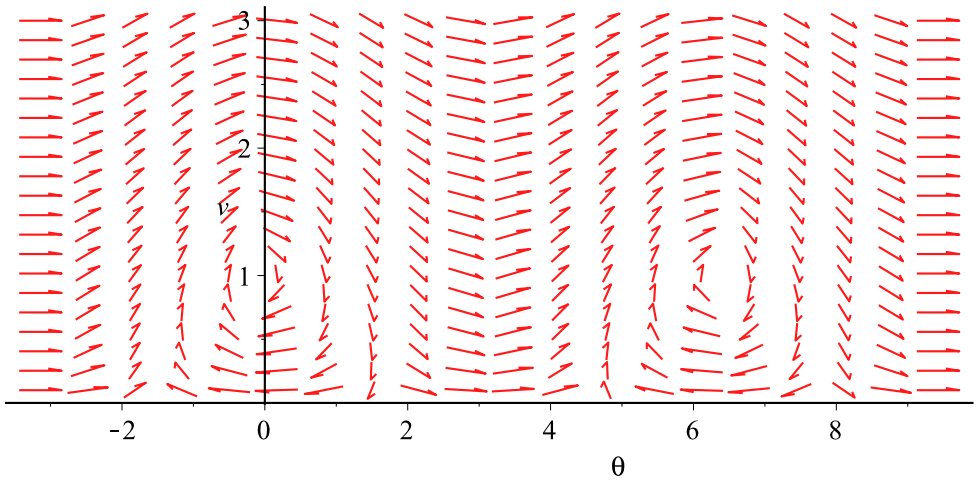
```
> phug:= R->[diff(theta(t),t) = (v(t)^2-cos(theta(t)))/v(t),
        diff(v(t),t)     = -sin(theta(t)) - R*v(t)^2 ];
```

$$phug := R \rightarrow \left[\frac{d}{dt}\,\theta(t) = \frac{v(t)^2 - \cos(\theta(t))}{v(t)},\ \frac{d}{dt}\,v(t) = -\sin(\theta(t)) - R\,v(t)^2 \right] \qquad (1)$$

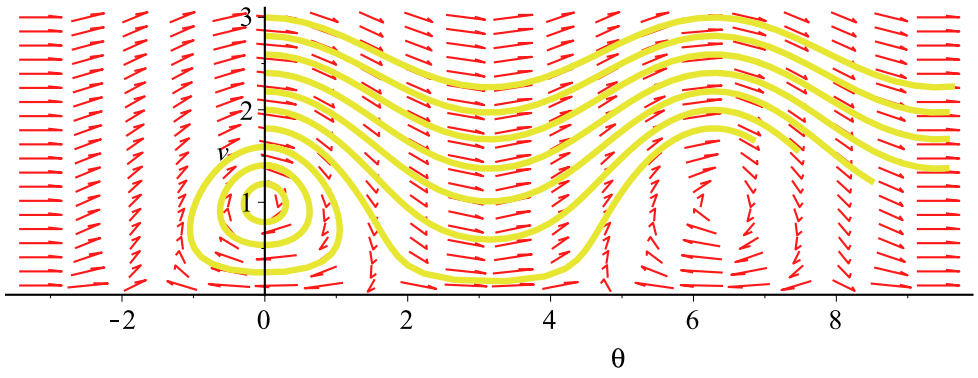
```
> with(DEtools):
```

Look at system in theta-v space with R=0

```
> DEplot(phug(0), [theta(t),v(t)], t=0..5,
        theta=-Pi..3*Pi, v=0.1..3);
```

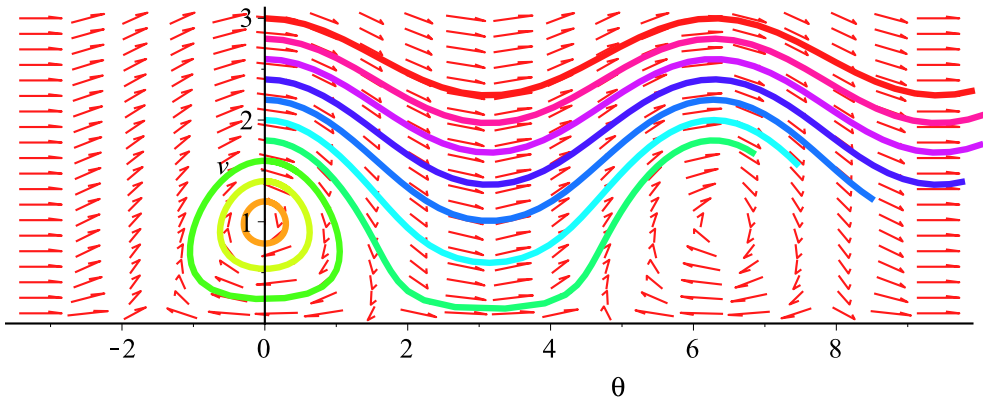



```
> DEplot(phug(0), [theta(t),v(t)], t=0..5,
        [ seq( [ theta(0)=0, v(0)=v0], v0=1..3, .2) ],
        theta=-Pi..3*Pi, v=0.1..3);
```

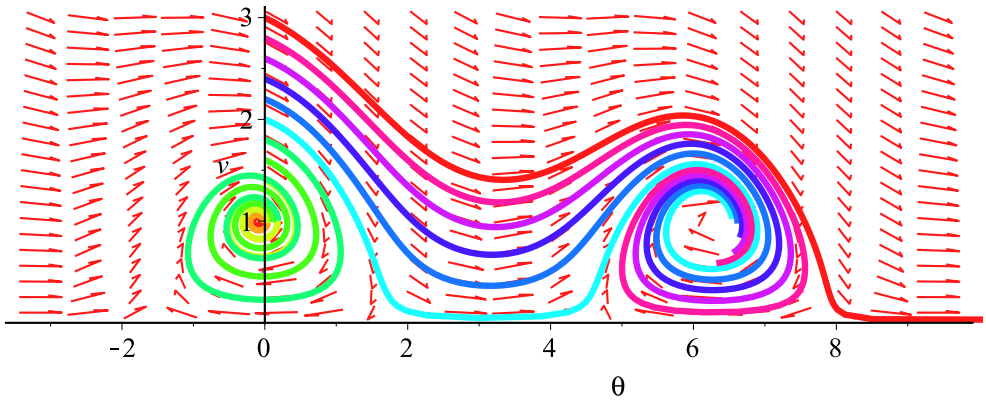


Yellow is ugly. Conditions go outside are dropped, must fix with obsrange.

```
> DEplot(phug(0), [theta(t),v(t)], t=0..5,
        [ seq( [ theta(0)=0, v(0)=v0], v0=1..3, .2) ],
        theta=-Pi..3*Pi, v=0.1..3,
        obsrange=false,
        linecolor=[seq(COLOR(HUE, h), h=0..1, .1)]);
```



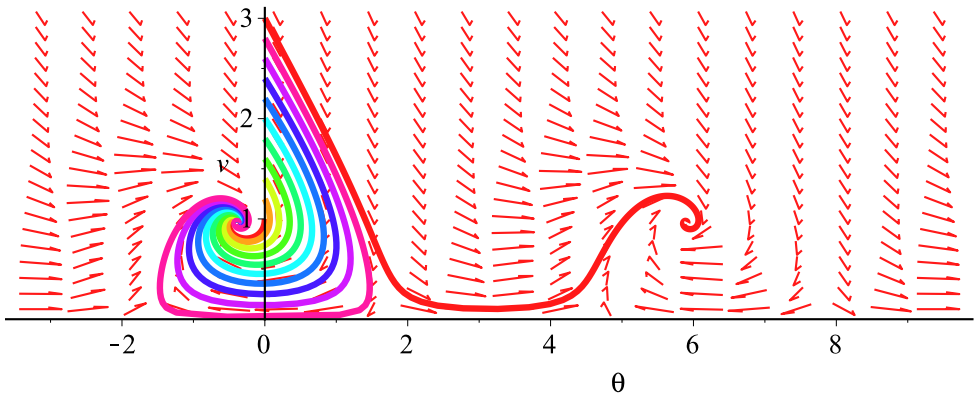

```
> DEplot(phug(0.1), [theta(t),v(t)], t=0..10,
       [ seq( [ theta(0)=0, v(0)=v0], v0=1..3, .2) ],
       theta=-Pi..3*Pi, v=0.1..3,
       obsrange=false,
       linecolor=[seq(COLOR(HUE, h), h=0..1, .1)],
       numpoints=200);
```



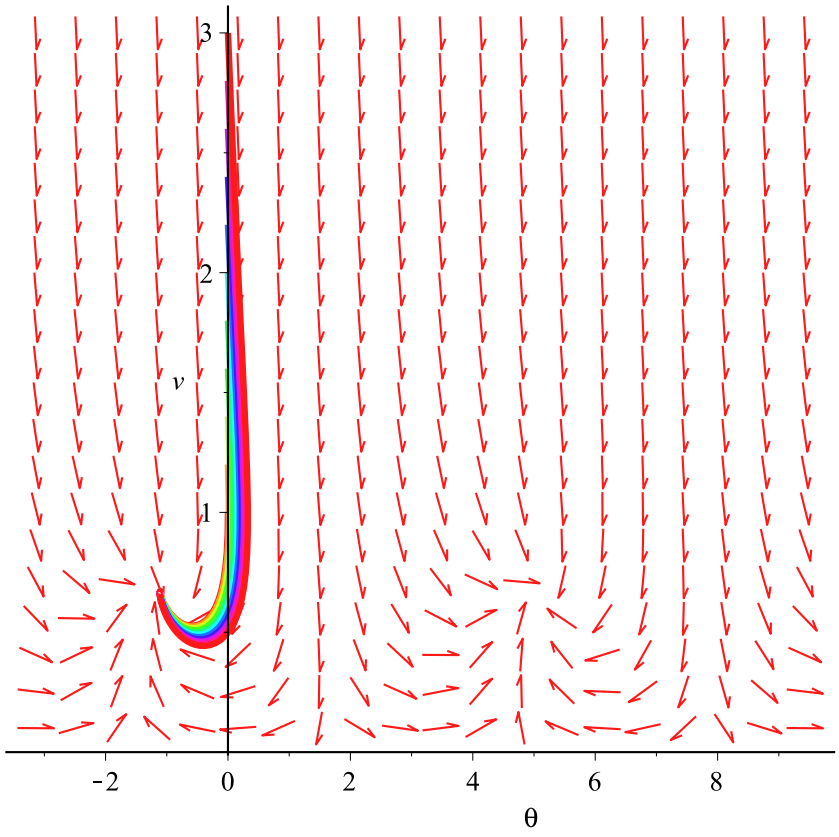

```
> phug(0.4);
```

$$\left[\frac{\mathrm{d}}{\mathrm{d}t}\,\theta(t) = \frac{v(t)^2 - \cos(\theta(t))}{v(t)},\ \frac{\mathrm{d}}{\mathrm{d}t}\,v(t) = -\sin(\theta(t)) - 0.4\,v(t)^2\right] \quad \textbf{(2)}$$

```
> DEplot(phug(0.4), [theta(t),v(t)], t=0..10,
       [ seq( [ theta(0)=0, v(0)=v0], v0=1..3, .2) ],
       theta=-Pi..3*Pi, v=0.1..3,
       obsrange=false,
       linecolor=[seq(COLOR(HUE, h), h=0..1, .1)],
       numpoints=200);
```

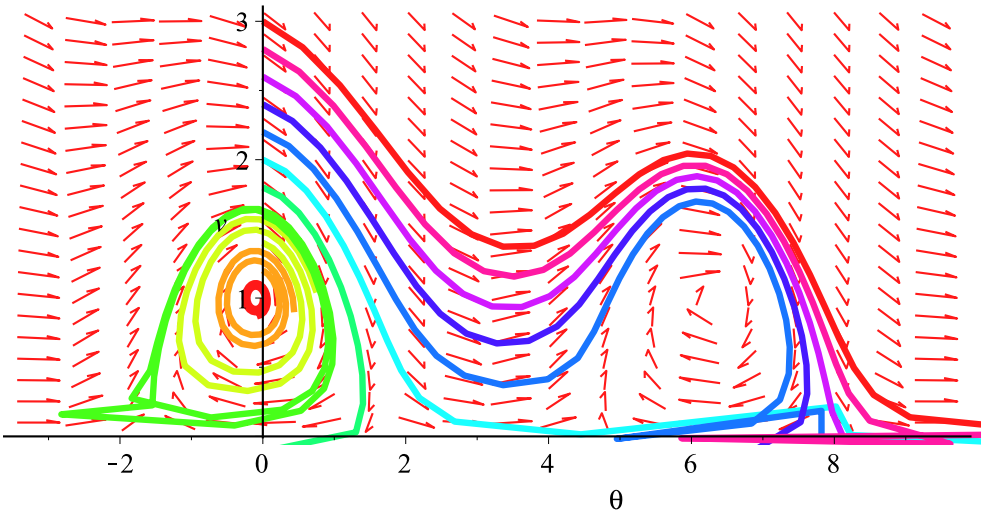



```
> DEplot(phug(2), [theta(t),v(t)], t=0..10,
       [ seq( [ theta(0)=0, v(0)=v0], v0=1..3, .2) ],
       theta=-Pi..3*Pi, v=0.1..3,
       obsrange=false,
       linecolor=[seq(COLOR(HUE, h), h=0..1, .1)],
       numpoints=200);
```

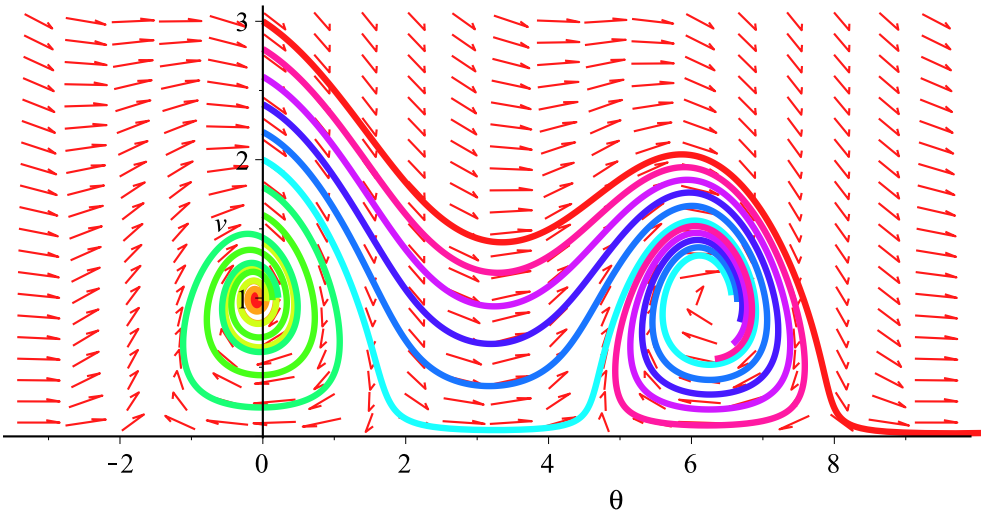


Do Euler's method for my solutions, with R=0.1

```
> DEplot(phug(0.1), [theta(t),v(t)], t=0..10,
       [ seq( [ theta(0)=0, v(0)=v0], v0=1..3, .2) ],
       theta=-Pi..3*Pi, v=0.1..3,
       obsrange=false,
       linecolor=[seq(COLOR(HUE, h), h=0..1, .1)],
       method=classical[foreuler]);
```

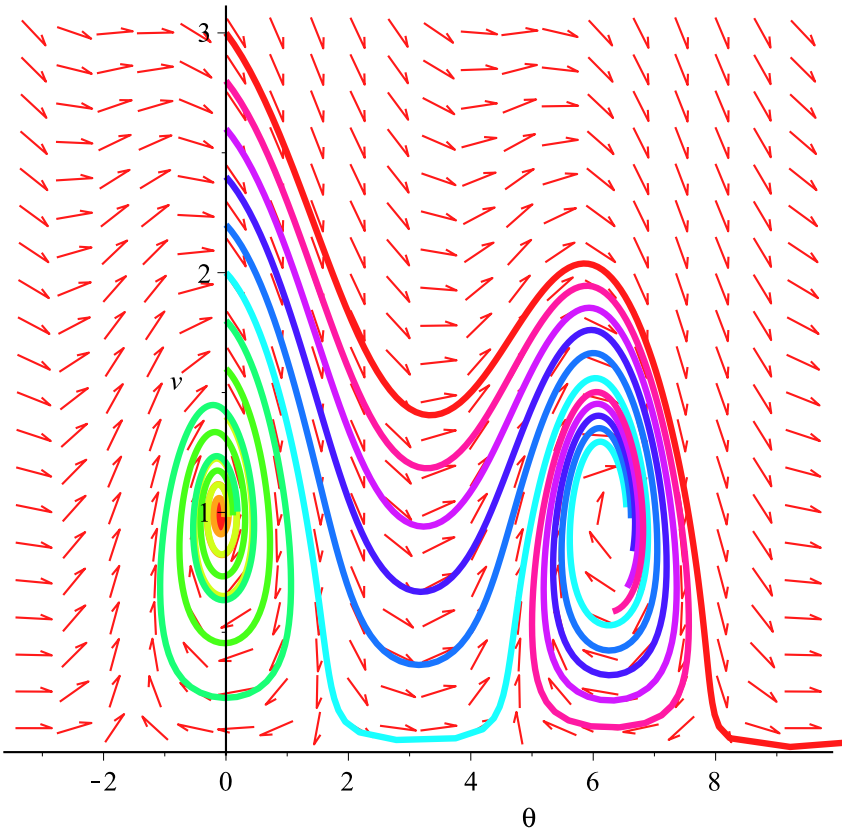



```
> DEplot(phug(0.1), [theta(t),v(t)], t=0..10,
       [ seq( [ theta(0)=0, v(0)=v0], v0=1..3, .2) ],
       theta=-Pi..3*Pi, v=0.1..3,
       obsrange=false,
       linecolor=[seq(COLOR(HUE, h), h=0..1, .1)],
       method=classical[foreuler], stepsize=0.01);
```

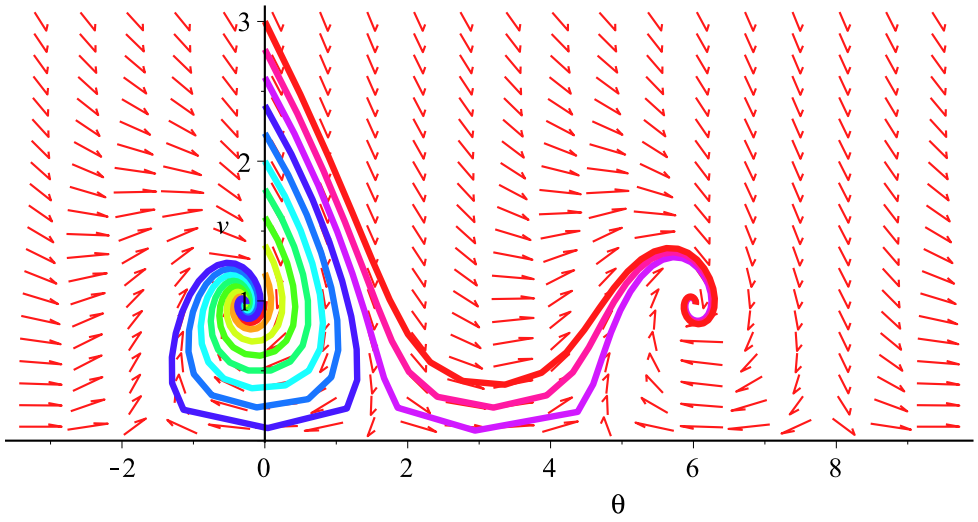



```
> DEplot(phug(0.1), [theta(t),v(t)], t=0..10,
```

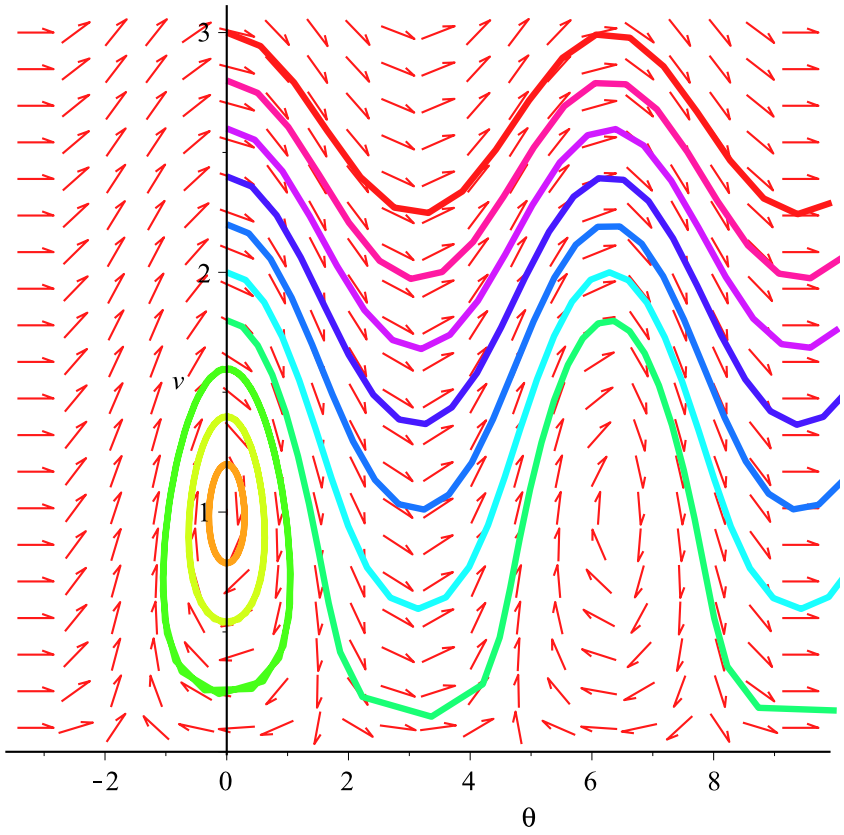
```
    [ seq( [ theta(0)=0, v(0)=v0], v0=1..3, .2) ],
    theta=-Pi..3*Pi, v=0.1..3,
    obsrange=false,
    linecolor=[seq(COLOR(HUE, h), h=0..1, .1)],
    method=classical[rk4], stepsize=0.05);
```




```
> movie:=
   [seq(
    DEplot(phug(R), [theta(t),v(t)], t=0..10,
        [ seq( [ theta(0)=0, v(0)=v0], v0=1..3, .2) ],
        theta=-Pi..3*Pi, v=0.1..3,
        obsrange=false,
        linecolor=[seq(COLOR(HUE, h), h=0..1, .1)]),
     R=0..1, .05)]:
> movie[4];
```




```
> with(plots):
> display(movie, insequence=true);
```



```
> phug(.2);
```

$$\left[\frac{d}{dt}\theta(t) = \frac{v(t)^2 - \cos(\theta(t))}{v(t)}, \frac{d}{dt}v(t) = -\sin(\theta(t)) - 0.2\,v(t)^2\right] \quad (3)$$

```
> xphug:= R-> [diff(theta(t),t) = (v(t)^2-cos(theta(t)))/v(t),
               diff(v(t),t)     = -sin(theta(t)) - R*v(t)^2 ,
               diff(x(t),t)     = v(t)*cos(theta(t)),
               diff(y(t),t)     = v(t)*sin(theta(t))];
```

$$xphug := R \rightarrow \left[\frac{d}{dt}\theta(t) = \frac{v(t)^2 - \cos(\theta(t))}{v(t)}, \frac{d}{dt}v(t) = -\sin(\theta(t)) - R\,v(t)^2, \frac{d}{dt}x(t) = v(t)\cos(\theta(t)), \frac{d}{dt}y(t) = v(t)\sin(\theta(t))\right] \quad (4)$$

```
> DEplot(xphug(0.25), [theta(t),v(t),x(t),y(t)], t=0..30,
       [ [theta(0)=0, v(0)=3, x(0)=0, y(0)=5]],
       theta=-Pi..3*Pi, v=0.1..3, x=0..10, y=0..7,
       obsrange=false,
       linecolor=[seq(COLOR(HUE, h), h=0..1, .1)],
       scene=[x,y], numpoints=200);
```