> $factor(x^2-1);$

$$(x-1)(x+1) \quad (1)$$

> $factor(x^2-5);$

$$x^2-5 \quad (2)$$

> $factor(x^2-5, complex);$

$$(x+2.236067977)(x-2.236067977) \quad (3)$$

> $factor(x^2-5, \mathrm{sqrt}(5));$

$$-(-x+\sqrt{5})(x+\sqrt{5}) \quad (4)$$

> $factor(x^2-b, \mathrm{sqrt}(5));$

$$x^2-b \quad (5)$$

> $factor(x^2-b, \mathrm{sqrt}(\mathrm{b}));$

$$-(x+\sqrt{b})(-x+\sqrt{b}) \quad (6)$$

> $solve(x^2=5, x);$

$$\sqrt{5}, -\sqrt{5} \quad (7)$$

> $(x-2)\cdot(x^4+3\cdot x-2);$

$$(x-2)(x^4+3x-2) \quad (8)$$

> $expand(\%);$

$$x^5+3x^2-8x-2x^4+4 \quad (9)$$

> $poly := \%;$

$$poly := x^5+3x^2-8x-2x^4+4 \quad (10)$$

> $solve(poly, x);$

$$2, \frac{1}{2}-\frac{1}{2}\mathrm{I}\sqrt{7}, \frac{1}{2}+\frac{1}{2}\mathrm{I}\sqrt{7}, \frac{1}{2}\sqrt{5}-\frac{1}{2}, -\frac{1}{2}-\frac{1}{2}\sqrt{5} \quad (11)$$

> $cracker := expand((x-2)\cdot(x^5+3\cdot x-2));$

$$cracker := x^6+3x^2-8x-2x^5+4 \quad (12)$$

> $\{solve(cracker, x)\};$

$$\{2, RootOf(\_Z^5+3\_Z-2, index=1), RootOf(\_Z^5+3\_Z-2, index=2), RootOf(\_Z^5+3\_Z-2, index=3), RootOf(\_Z^5+3\_Z-2, index=4), RootOf(\_Z^5+3\_Z-2, index=5)\} \quad (13)$$

> $evalf(\%)$

$$\{0.632834520242152, 2., -1.06488575452018-0.950546034963830\,\mathrm{I}, -1.06488575452018+0.950546034963830\,\mathrm{I}, 0.748468494399101-0.995433954467932\,\mathrm{I}, 0.748468494399101+0.995433954467932\,\mathrm{I}\} \quad (14)$$

> $fsolve(cracker);$

$$0.6328345202, 2. \quad (15)$$

>

>

> $data := [[1,1],[2,3],[3,-1],[4,0],[6,2]];$

$$data := [[1,1],[2,3],[3,-1],[4,0],[6,2]] \quad (16)$$

> $CurveFitting[PolynomialInterpolation]((\mathbf{16}), x)$

$$-\frac{59}{120}x^4+\frac{27}{4}x^3-\frac{749}{24}x^2+\frac{223}{4}x-\frac{149}{5} \quad \textbf{(17)}$$

last time, set up f(1)=1, f(2)=3... etc. & solved.

> $f := x \to a \cdot x^4 + b \cdot x^3 + c \cdot x^2 + d \cdot x + e;$

$$f := x \to a\,x^4 + b\,x^3 + c\,x^2 + d\,x + e \quad \textbf{(18)}$$

> $seq(f(data[i][1]) = data[i][2], i = 1..nops(data));$

$$a+b+c+d+e=1, 16\,a+8\,b+4\,c+2\,d+e=3, 81\,a+27\,b+9\,c+3\,d+e=-1, 256\,a + 64\,b+16\,c+4\,d+e=0, 1296\,a+216\,b+36\,c+6\,d+e=2 \quad \textbf{(19)}$$

nops counts elements in a list

> $nops(data);$

$$5 \quad \textbf{(20)}$$

> $nops([1, 2, 3, [4, 5, 6, 7], 8, shoe]);$

$$6 \quad \textbf{(21)}$$

> $solve(\{seq(f(data[i][1]) = data[i][2], i = 1..nops(data))\}, \{a, b, c, d, e\})$
> $;$

$$\left\{a=-\frac{59}{120}, b=\frac{27}{4}, c=-\frac{749}{24}, d=\frac{223}{4}, e=-\frac{149}{5}\right\} \quad \textbf{(22)}$$

> $data;$

$$[[1, 1], [2, 3], [3, -1], [4, 0], [6, 2]] \quad \textbf{(23)}$$

> $f(data[3][1]) = data[3][2]$

$$81\,a+27\,b+9\,c+3\,d+e=-1 \quad \textbf{(24)}$$

> $eqns := \{seq(f(data[i][1]) = data[i][2], i = 1..nops(data))\}$

$$eqns := \{a+b+c+d+e=1, 16\,a+8\,b+4\,c+2\,d+e=3, 81\,a+27\,b+9\,c+3\,d+e= -1, 256\,a+64\,b+16\,c+4\,d+e=0, 1296\,a+216\,b+36\,c+6\,d+e=2\} \quad \textbf{(25)}$$

> $coef := solve(\%, \{a, b, c, d, e\});$

$$coef := \left\{a=-\frac{59}{120}, b=\frac{27}{4}, c=-\frac{749}{24}, d=\frac{223}{4}, e=-\frac{149}{5}\right\} \quad \textbf{(26)}$$

> $f(x);$

$$a\,x^4+b\,x^3+c\,x^2+d\,x+e \quad \textbf{(27)}$$

want to plug coef into f(x) to get a polynomial in x

> $subs(coef, f(x));$

$$-\frac{59}{120}x^4+\frac{27}{4}x^3-\frac{749}{24}x^2+\frac{223}{4}x-\frac{149}{5} \quad \textbf{(28)}$$

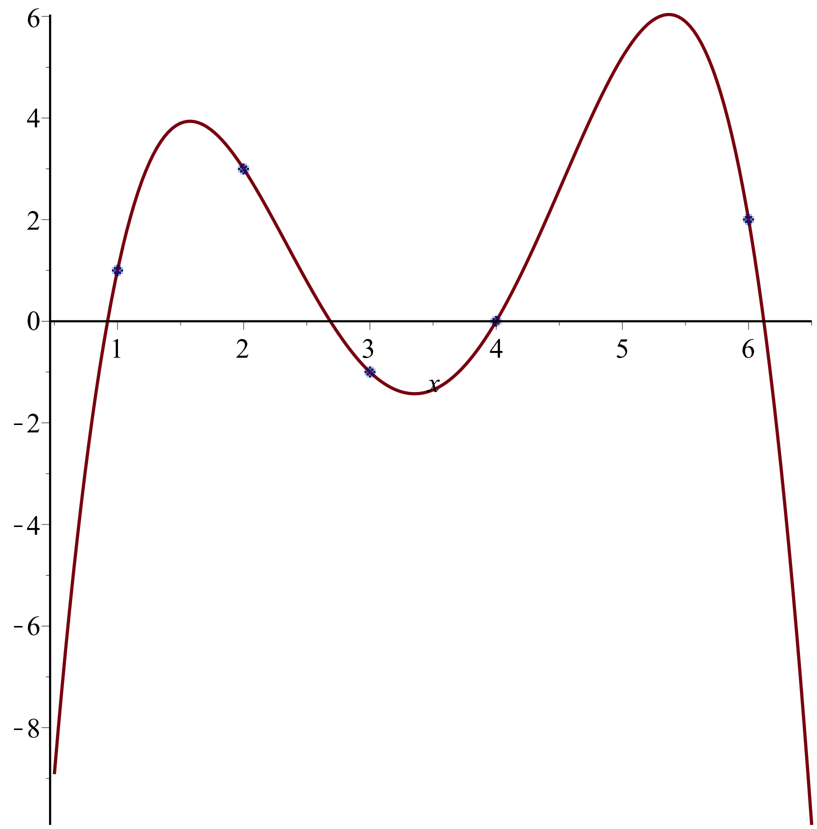
> $subs(rabbit = cow, rabbit \cdot dog + cat \cdot rabbit^2)$

$$cow\ dog + cat\ cow^2 \quad \textbf{(29)}$$

> $g := unapply(subs(coef, f(x)), x);$

$$g := x \to -\frac{59}{120}x^4+\frac{27}{4}x^3-\frac{749}{24}x^2+\frac{223}{4}x-\frac{149}{5} \quad \textbf{(30)}$$

> $plot([g(x), data], x = 0.5..6.5, style = [line, point]);$



> *CurveFitting[PolynomialInterpolation](data, x)*

$$-\frac{59}{120}x^4+\frac{27}{4}x^3-\frac{749}{24}x^2+\frac{223}{4}x-\frac{149}{5} \quad \textbf{(31)}$$

> *g(x);*

$$-\frac{59}{120}x^4+\frac{27}{4}x^3-\frac{749}{24}x^2+\frac{223}{4}x-\frac{149}{5} \quad \textbf{(32)}$$

> *f(x);*

$$a\,x^4+b\,x^3+c\,x^2+d\,x+e \quad \textbf{(33)}$$

> *indets(f(1));*

$$\{a, b, c, d, e\} \quad \textbf{(34)}$$

> *solve({seq(f(data[i][1]) = data[i][2], i = 1 ..nops(data)) }, indets(f(1)));*

$$\left\{a=-\frac{59}{120}, b=\frac{27}{4}, c=-\frac{749}{24}, d=\frac{223}{4}, e=-\frac{149}{5}\right\} \quad \textbf{(35)}$$

> *subs(solve({seq(f(data[i][1]) = data[i][2], i = 1 ..nops(data)) }, indets(f(1))), f(x));*

$$-\frac{59}{120}x^4+\frac{27}{4}x^3-\frac{749}{24}x^2+\frac{223}{4}x-\frac{149}{5} \quad \textbf{(36)}$$

> *g := unapply(subs(solve({seq(f(data[i][1]) = data[i][2], i = 1 ..nops(data)) },*
> *indets(f(1))), f(x)), x);*

$$g := x \to -\frac{59}{120}x^4 + \frac{27}{4}x^3 - \frac{749}{24}x^2 + \frac{223}{4}x - \frac{149}{5} \quad (37)$$

>

>

> $dat := [[0, 0], [1, 0], [2, 0], [3, 0], [4, 0]];$

$$dat := [[0, 0], [1, 0], [2, 0], [3, 0], [4, 0]] \quad (38)$$

> $subs(\, solve(\{seq(\, f(dat[i][1]) = dat[i][2],\, i = 1..nops(dat))\},\, indets(\, f(1))),\, f(x))$

$$0 \quad (39)$$

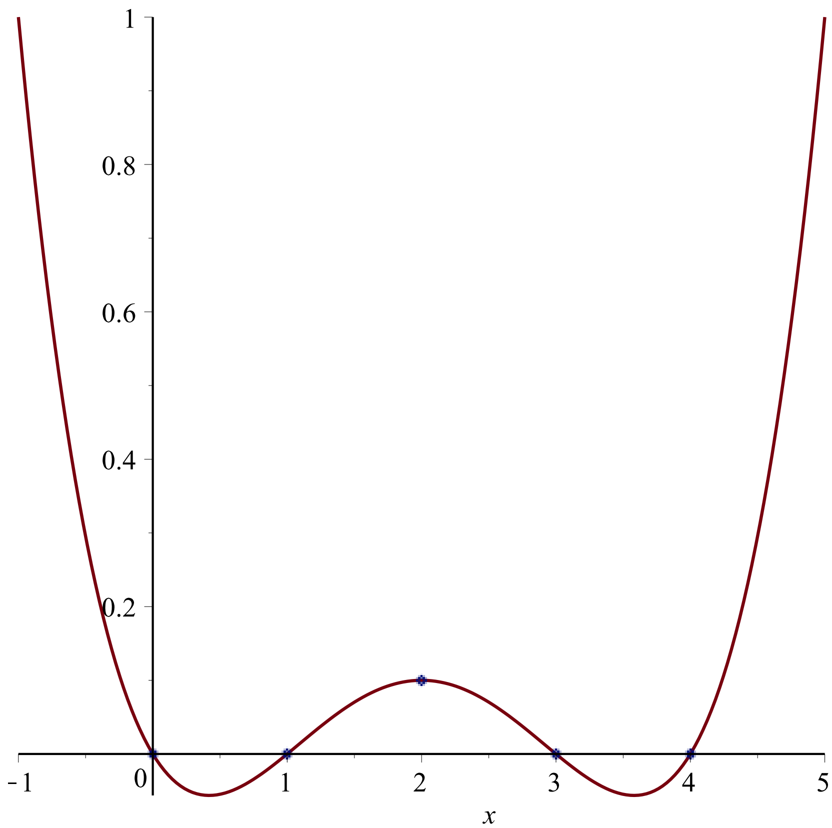
> $dat := \left[[0, 0], [1, 0], \left[2, \frac{1}{10}\right], [3, 0], [4, 0]\right];$

$$dat := \left[[0, 0], [1, 0], \left[2, \frac{1}{10}\right], [3, 0], [4, 0]\right] \quad (40)$$

> $CurveFitting[PolynomialInterpolation](\, \mathbf{(40)},\, x\,)$

$$\frac{1}{40}x^4 - \frac{1}{5}x^3 + \frac{19}{40}x^2 - \frac{3}{10}x \quad (41)$$

> $plot([h, dat], x = -1..5, style = [line, point]);$



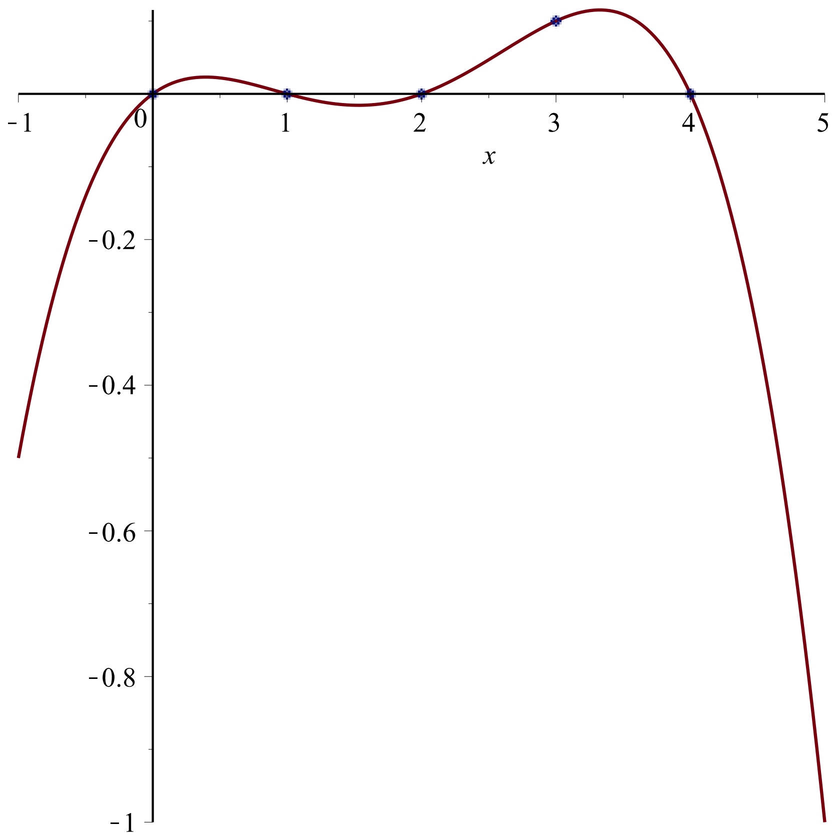
> $dat := \left[[0,0],[1,0],[2,0],\left[3,\frac{1}{10}\right],[4,0]\right];$

$$dat := \left[[0,0],[1,0],[2,0],\left[3,\frac{1}{10}\right],[4,0]\right] \quad \textbf{(42)}$$

> $j := CurveFitting[PolynomialInterpolation](\textbf{(42)}, x)$

$$j := -\frac{1}{60}x^4 + \frac{7}{60}x^3 - \frac{7}{30}x^2 + \frac{2}{15}x \quad \textbf{(43)}$$

> $plot([j, dat], x=-1..5, style=[line, point]);$



> $dat := \left[seq([i,0], i=0..3), \left[\frac{7}{2}, 1\right], seq([i,0], i=4..20)\right];$

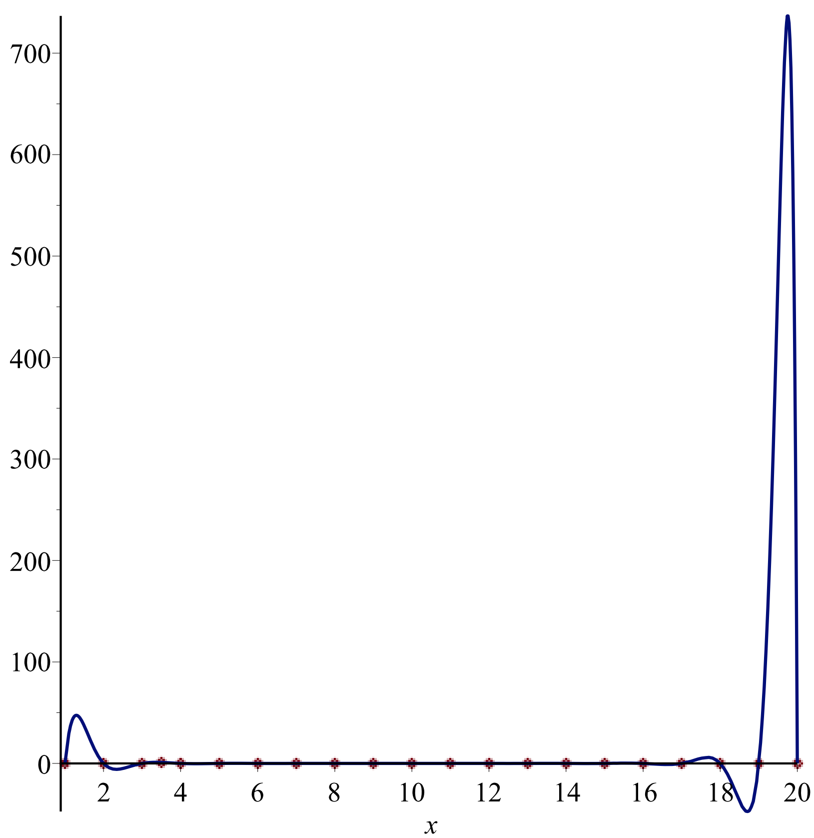
$$dat := \left[[0,0],[1,0],[2,0],[3,0],\left[\frac{7}{2},1\right],[4,0],[5,0],[6,0],[7,0],[8,0],[9,0],[10,0],[11,0],[12,0],[13,0],[14,0],[15,0],[16,0],[17,0],[18,0],[19,0],[20,0]\right] \quad \textbf{(44)}$$

> $ick := CurveFitting[PolynomialInterpolation](dat, x);$

$$ick := -\frac{140823000329855602524 16}{555496479891478125}x^5 + \frac{14317433373184753664}{352696177708875}x^4 \quad \textbf{(45)}$$

$$
\begin{aligned}
&- \frac{22384111941562295 9104}{58817274341450625} x^{7} + \frac{2120063252254687232}{186721505845875} x^{6} \\
&- \frac{2859119246900455276544}{65672028289392525} x^{3} + \frac{383696126068391936}{13898841966009} x^{2} \\
&- \frac{409242085621239906304}{2058604601950771875} x^{9} - \frac{270408429338624}{65571097370625} x^{11} + \frac{663844166626705408}{676060624614375} x^{8} \\
&- \frac{549755813888}{71645805} x - \frac{2097152}{6649292864300993156 25} x^{21} + \frac{4194304}{1058088533126625} x^{18} \\
&- \frac{2097152}{32254634316279375} x^{19} + \frac{4194304}{6332659870762850625} x^{20} - \frac{52164898586624}{411720920390154375} x^{15} \\
&+ \frac{4924112896}{933607529229375} x^{16} - \frac{16487809024}{98028790569084375} x^{17} + \frac{30710555475968}{71815963786875} x^{12} \\
&- \frac{806973429776384}{22622028592865625} x^{13} + \frac{267168776192}{112032903507525} x^{14} + \frac{459453837082624}{14363192757375} x^{10}
\end{aligned}
$$

```
> plot([dat, ick], x = 1 ..20, style = [point, line]);
```



it wiggles a lot far away from x=7/2 cuz polynomials don't have a lot of freedom.

What is going wrong below

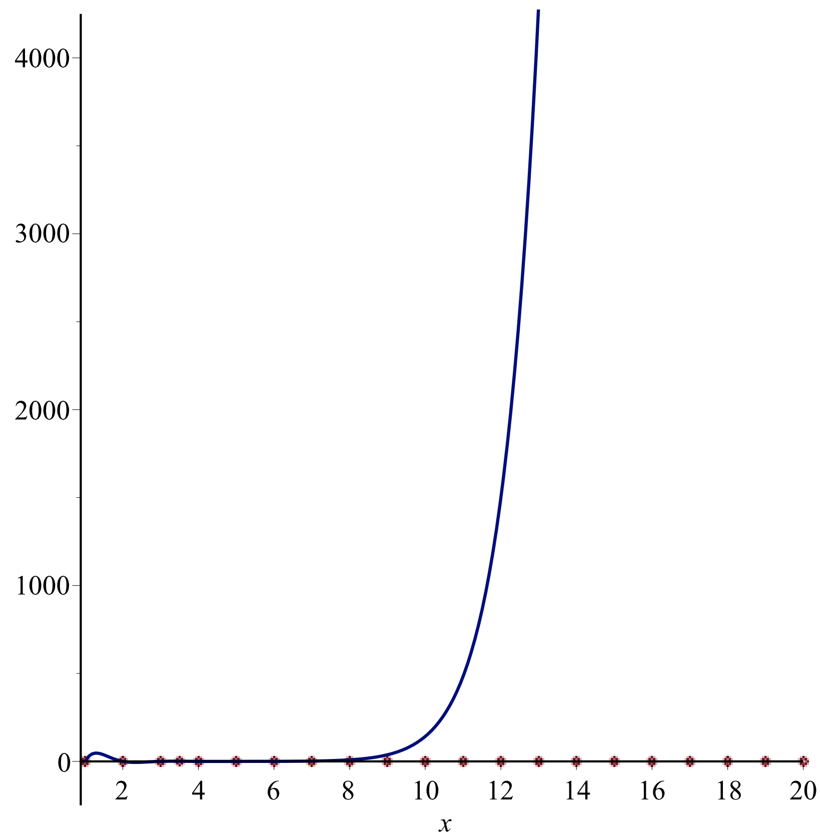
> $[seq([i,0], i=0..3), [3.5, 1], seq([i,0], i=4..20)];$

$$[[0,0],[1,0],[2,0],[3,0],[3.5,1],[4,0],[5,0],[6,0],[7,0],[8,0],[9,0],[10,0],[11,0],[12,0],[13,0],[14,0],[15,0],[16,0],[17,0],[18,0],[19,0],[20,0]] \quad (46)$$

> $bad := CurveFitting[PolynomialInterpolation]((\mathbf{46}), x)$

$$bad := -25362.94646\,x^5 + 40612.15102\,x^4 - 3807.849218\,x^7 + 11360.03455\,x^6 - 43554.15055\,x^3 + 27616.88766\,x^2 - 198.9283200\,x^9 - 4.127131059\,x^{11} + 982.5318798\,x^8 - 7676.009406\,x - 3.158883612\;10^{-15}\,x^{21} + 3.969204231\;10^{-9}\,x^{18} - 6.510889144\;10^{-11}\,x^{19} + 6.633063652\;10^{-13}\,x^{20} - 0.0001268342811\,x^{15} + 0.000005280298447\,x^{16} - 1.683987729\;10^{-7}\,x^{17} + 0.4279915138\,x^{12} - 0.03570472284\,x^{13} + 0.002387091271\,x^{14} + 32.01142124\,x^{10} \quad (47)$$

> $plot([dat, bad], \mathrm{x} = 1..20, \mathrm{style} = [\mathrm{point}, \mathrm{line}]);$

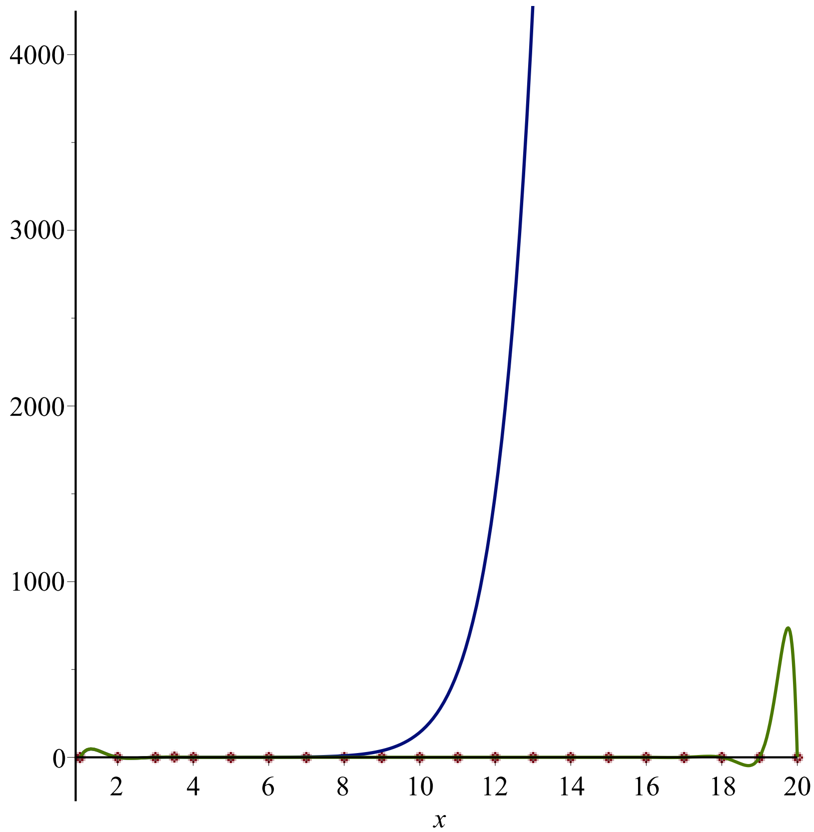


> $Digits := 100;$

$$Digits := 100 \quad (48)$$

> $better := CurveFitting[PolynomialInterpolation]((\mathbf{46}), x):$

> $plot([dat, bad, better], \mathrm{x} = 1..20, \mathrm{style} = [\mathrm{point}, \mathrm{line}, \mathrm{line}]);$


4000
3000
2000
1000
0
2
4
6
8
10
12
14
16
18
20
x

>