MAT 331, Spring 2012

Project 2b: Gliders and Differential Equations

Due Tuesday, April 17

As we have discussed in class, the flight of a glider can be approximately described by the system

$$\frac{dv}{dt} = -\sin\theta - Rv^2 \qquad \frac{d\theta}{dt} = \frac{v^2 - \cos\theta}{v}$$

where $v > 0$ is the speed of the glider, and θ is the angle the nose makes with the horizontal. The $\sin\theta$ and $\cos\theta$ terms represent the effects of gravity, and the Rv^2 and v^2 account for drag and lift, respectively. The parameter R adjusts the strength of the drag on the plane due to air resistance. In this project, we will fix the drag constant as $R = 0.1$.

The goal of this project is to answer the following questions:

1. When the glider is launched with an initial angle of 0, for what range of initial velocities (to the nearest 0.05) will the glider make exactly one loop?

2. If the glider is launched from a height of 2 units, can you find an initial velocity and angle so that the glider lands exactly 20 horizontal units away? (There are many such initial conditions).

3. If the glider is launched from a height of 2 units with an inital velocity of 2, what angle (to the nearest hundredth of a radian) allows the glider to go the furthest before it hits the ground? What if the initial velocity is 0.5 instead?

4. Is it possible to arrange it, by varying both the initial velocity and angle but keeping the initial height at 2, so that the glider "lands gently" instead of crashing? (That is, so that when the height is 0, the glider is approximately horizontal.) If it isn't possible, give a justification. (This question is a bit harder than the others, and is optional. It would be nice if you would at least try.)

For all the questions, you should give sufficient justification of your answers. Please remember that half of the grade of your project depends on the clarity of your exposition. Include *relevant* graphs in your explanation.

When doing part 3 above (and perhaps the other parts), you should modify the bisection routine we wrote in class to zero in on the maximum, rather than just finding it by trial and error. In your write-up, describe how your algorithm works.