22. (*expires 4/20*) Write a function to compute the n-th partial sum of the alternating series

$$1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} - \frac{1}{36} + \dots$$

Specifically, the procedure takes n as input, and returns the sum of the first n terms, using a loop.

23. (*expires 4/20*) Investigate the limit of $\sin(x)/x$ as $x \to 0$. More specifically, define

$$f(x) = \frac{\sin x}{x}$$

and demonstrate that $\lim_{x\to 0} f(x) = 1$ as follows.

Write a function that takes ϵ as input, and calculates an integer N so that $|f(1/N) - 1| < \epsilon$, using a while loop.

Specifically, compute $f(1), f(1/2), f(1/3), \dots$ until $|f(1/N) - 1| < \epsilon$, and return N.

24. (*expires 4/27*) Calculate the determinant of the square Vandermonde matrix

$$V = \begin{pmatrix} 1 & \alpha_1 & \alpha_1^2 & \alpha_1^3 \\ 1 & \alpha_2 & \alpha_2^2 & \alpha_2^3 \\ 1 & \alpha_3 & \alpha_3^2 & \alpha_3^3 \\ 1 & \alpha_4 & \alpha_4^2 & \alpha_4^3 \end{pmatrix}$$

The determinant of an $n \times n$ Vandermonde matrix can be expressed as

$$\det(V) = \prod_{1\le i<j\le n} (\alpha_j - \alpha_i)$$

Write a procedure `pdetVandermonde` which takes a list $\alpha = [\alpha_1, \alpha_2, \dots, \alpha_n]$ and calculates its determinant using the formula above. This can be done using a double loop (note that $i < j$).

Test this procedure with the list $[1, 2, 3, 4]$ corresponding to the matrix

$$V = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \end{pmatrix}$$

Given a vector α, the `Maple` function `LinearAlgebra[VandermondeMatrix]`(α) will create a Vandermonde matrix as above.

For comparison (not as part of the procedure), convert your list to a vector, then use the command `VandermondeMatrix` to generate the corresponding Vandermonde matrix V, and compare your answer to the one given by `Determinant(V)`.

25. (*expires 4/27*) Write a function that takes an integer N as input, and outputs the first N terms of the Fibonacci sequence given by

$$F_1 = 1, \quad F_2 = 1, \quad F_n = F_{n-1} + F_{n-2} \text{ for } n > 2.$$

Test this with $N = 35$.