

17. (expires 3/15) (a last gasp for curve-fitting problems) The file `bdata.txt` on the class website defines a list called `bdata`. This is a list of 54 points which approximate a line, but several of the points are quite far away from the rest.

Using the robust fitting method discussed in class on February 28, or in chapter 7 of the notes, with the objective functions

$$R(m, b, data) = \sum_{i=1}^n \ln(1 + (mx_i + b - y_i)^2) \quad \text{and} \quad S(m, b, data) = \sum_{i=1}^n 1 - e^{-(mx_i + b - y_i)^2}.$$

Determine the best lines for the data using regular least squares, as well as those given by R and S above. Plot the data and the lines you found on the same plot, both in a view that shows all the data points, and in another which restricts the y values to those for which there is a data point within 10 units of a fit line (ie, $0 < y < 35$).

18. (expires 3/15) Find all the solutions to the differential equation

$$\frac{dx}{dt}(t) = -2x(t), \quad t \in \mathbb{R}.$$

Among them, single out the one for which $x(0) = 3$. [Hint: read the help page for `dsolve`, or just do it in your head. It is that easy.]

19. (expires 3/15) Have Maple find analytic solutions to the following system of differential equations,

$$\begin{cases} y''(t) - z(t) = e^t, \\ z'(t) - y(t) = 0, \end{cases}$$

with initial conditions: $y(0) = 1$, $y'(0) = 0$, $z(0) = k$. Let us denote the solutions by $y_k(t)$, $z_k(t)$ (since they depend on the parameter k).

For k taking all integer values from -10 to 10, and $t \in [-4, 2]$, plot the functions y_k in blue, and the functions z_k in red, all on the same graph. (Yes, you will then have 42 functions plotted on the same graph.) [This is certainly a case when you don't want to retype the functions that Maple finds. You will almost certainly need to read the help page for `dsolve`. I also found `subs`, `unapply`, and `seq` useful.]

20. (expires 3/15) For the functions $y_k(t)$ and $z_k(t)$ found in the previous problem, plot the parametric curves $\varphi_k(t) = [y_k(t), z_k(t)]$ for integer values of k between -5 and 5 and $-6 < t < 4$ on the same graph. Use the `view` option of plot to only show what lies in the region $-10 < y < 10$, $-10 < z < 10$, and use a sequence of colors so that each solution is a different color. [HINT: you might find something like `seq(COLOR(HUE, i/11), i=0..10)` useful for the latter.]

21. (expires 3/15) Find all the fixed points of the system

$$\begin{cases} \dot{x} = x^2 + y, \\ \dot{y} = x(y^2 - 1), \end{cases}$$

where a “fixed point” is a solution for which both $x(t)$ and $y(t)$ are constant. For each of these solutions you find, describe the behavior of the solutions that have initial conditions nearby. You can use Maple to figure out what happens for nearby points, or you can use more mathematical methods.

NOTE: The fact that there are various notations for differential equations is purely intentional.