\[
\text{with(DEtools):} \\
\text{phug}:=\left[ \frac{D(\theta(t))}{v(t)} = v(t) - \frac{\cos(\theta(t))}{v(t)}, \text{D}(v)(t) = -\sin(\theta(t)) - R \cdot v(t)^2 \right]; \\
\text{R}:=0.3; \\
\text{stuff}:=[\theta(t), v(t)], t=-1..20, \\
\text{theta}=-\pi..2\pi, v=0..3, \\
[\text{seq}([\theta(0)=0, v(0)=i], i=1..3, 0.2)], \\
\text{linecolor}=[\text{seq(COLOR(HUE,i), i=-0.1..0.1, 0.01)}], \text{stepsize}=0.05: \\
\text{DEplot(phug, stuff, scene=[theta,v])}; \\
\]
\[ \text{stuff} := \{\theta(t), v(t)\}, \quad t = -1..20, \]
\[ \quad \theta = 1..2, \quad v = 0..0.1, \]
\[ \quad \{\theta(0) = \pi/2 + i, \quad v(0) = 0.1\}, i = -0.1..0.1, 0.01\}, \]
\[ \text{linecolor} = \{\text{seq}(\text{COLOR}(\text{HUE}, i), i = -0.1..0.1, 0.01)\}, \quad \text{stepsize} = 0.005: \]
\[ \text{DEplot(phug, stuff, scene=[\theta, v])}; \]
> stuff:=[theta(t), v(t)], t=-1..20,
  theta=-Pi..2*Pi, v=0..3,
  [seq([theta(0)=0, v(0)=i], i=1.8..2, 0.05)],
  linecolor=[seq(COLOR(HUE,i), i=1.8..2, 0.05)], stepsize=0.05:
DEplot(phug, stuff, scene=[theta, v]);
> stuff:=[theta(t), v(t)], t=-5..10, 
  theta=-0..Pi, v=0..2, 
  [[theta(0)=Pi/2, v(0)=0.01], seq([theta(0)=0, v(0)=i],i=1.8..2, 0.05)], 
  linecolor=[black,seq(COLOR(HUE,i),i=1.8..2,0.05)], obsrange=false,stepsize=0.05:
DEplot(phug, stuff, scene=[theta,v]);
\[
\text{\texttt{DEplot(phug, stuff, scene=[\theta, v]);}}
\]
\[
\theta = [\theta(t), v(t)], \ t = -5..10, \\
\theta = -0..\pi, \ v = 0..2, \\
[[\theta(0) = \pi/2, v(0) = 0], \text{seq}([[\theta(0) = 0, v(0) = i], i = 1.8..2, 0.05))], \\
\text{linecolor} = [\text{black}, \text{seq}(	ext{COLOR(HUE, i)}, i = 1.8..2, 0.05)], \ \text{obsrange} = \text{false, stepsize} = 0.005:\
\text{DEplot(phug, stuff, scene} = [\theta, v]);
\]

Warning, plot may be incomplete, the following error(s) were issued:

cannot evaluate the solution past the initial point, problem may be complex, initially singular or improperly set up
\[
\text{DEplot(phug, stuff, scene=}[\text{theta, v}]));
\]
\[
\begin{align*}
\text{vphug} &:= \left[ D(\theta)(t) = v(t)^2 - \cos(\theta(t)), \\
D(v)(t) & = (-\sin(\theta(t)) - R^* v(t)^2) v(t) \right]; \\
\text{stuff} &:= \{ \theta(t), v(t) \}, t = -5..10, \\
\text{theta} &:= \pi/2..\pi, v = 0..2, \\
\text{linecolor} &:= \text{black}, \text{seq}(\text{COLOR(HUE, i)}, i = 1.8..2, 0.05), \\
\text{obsrange} &:= \text{false}, \text{stepsize} = 0.05; \\
\text{DEplot(vphug, stuff, scene=[theta, v]);}
\end{align*}
\]

Warning, plot may be incomplete, the following errors(s) were issued:
- cannot evaluate the solution further left of -0.69973011, probably a singularity
- cannot evaluate the solution further left of -0.64930540, probably a singularity
- cannot evaluate the solution further left of -0.60461454, probably a singularity
Warning, plot may be incomplete, the following errors(s) were issued:
cannot evaluate the solution further left of -.56476163, probably a singularity
Warning, plot may be incomplete, the following errors(s) were issued:
cannot evaluate the solution further left of -.52902775, probably a singularity

\[
F := (\theta, v) \rightarrow [v^2 - \cos(\theta), v(-\sin(\theta) - 3v^2)]
\]

\[
\text{with(VectorCalculus)};
\]  
\[
\text{Jacobian}(F(\theta, v), [\theta, v]);
\]

\[
\begin{bmatrix}
\sin(\theta) & 2v \\
-vcos(\theta) & -\sin(\theta) - 3v^2
\end{bmatrix}
\]

\[
eval(%, \{\theta=\pi/2, v=0\});
\]

\[
\begin{bmatrix}
1 & 0 \\
0 & -1
\end{bmatrix}
\]

\[
\text{Jacobian}(F(\theta, v), [\theta, v]);
\]
\[ \text{eval}(\%, \{\theta = -\pi/2, \ v = 0\}); \]

\[
\begin{bmatrix}
-1 & 0 \\
0 & 1
\end{bmatrix}
\]

Want plane to stop when \( y(t) = 0 \).

\[ \text{xphug} := [\ D(\theta)(t) = v(t) - \cos(\theta(t))/v(t),
\quad D(v)(t) = -\sin(\theta(t)) - R \cdot v(t)^2,
\quad D(x)(t) = v(t) \cdot \cos(\theta(t)),
\quad D(y)(t) = v(t) \cdot \sin(\theta(t)) ]; \]

\[ xphug := \begin{bmatrix}
D(\theta)(t) = v(t) - \frac{\cos(\theta(t))}{v(t)},\ D(v)(t) = -\sin(\theta(t)) - 0.3 \cdot v(t)^2,\ D(x)(t) \\
= v(t) \cdot \cos(\theta(t)),\ D(y)(t) = v(t) \cdot \sin(\theta(t)) 
\end{bmatrix} \]

\[ \text{crashy} := [\ D(\theta)(t) = v(t) - \cos(\theta(t))/v(t),
\quad D(v)(t) = -\sin(\theta(t)) - R \cdot v(t)^2,
\quad D(x)(t) = v(t) \cdot \cos(\theta(t)),
\quad D(y)(t) = \text{piecewise}(y(t) > 0, v(t) \cdot \sin(\theta(t)), 0)]; \]

\[ \text{crashy} := \begin{bmatrix}
D(\theta)(t) = v(t) - \frac{\cos(\theta(t))}{v(t)},\ D(v)(t) = -\sin(\theta(t)) - 0.1 \cdot v(t)^2,\ D(x)(t) \\
= v(t) \cdot \cos(\theta(t)),\ D(y)(t) = \begin{cases} v(t) \cdot \sin(\theta(t)) & 0 < y(t) \\ 0 & \text{otherwise} \end{cases} 
\end{bmatrix} \]

\[ \text{with(plots)}: \]

\[ \text{R} := 1; \]

\[ \text{stuff} := [\theta(t), v(t), x(t), y(t)], \ t = -1..20,\]

\[ \theta = -\pi..4 \cdot \pi,\ v = 0..3,\ x = -3..10,\ y = -3..3,\]

\[ \text{seq}([\theta(0) = 0, \ v(0) = i, \ x(0) = 0, \ y(0) = 0], i = 1..3, 0.2) ];\]

\[ \text{linecolor} = \text{seq}([\text{COLOR}(HUE, i), i = 0..1, 1]), \text{stepsize} = 0.05; \]

\[ \text{display} (\text{array}(\ [\text{DEplot}(\text{crashy}, \text{stuff}, \text{scene} = [\theta, v]),
\quad \text{DEplot}(\text{crashy}, \text{stuff}, \text{scene} = [x, y]) \ ])); \]

\[ R := 1 \]
\[
\text{crashy} := \left[
\begin{align*}
\frac{d}{dt} \theta(t) &= \begin{cases} 
  v(t) - \frac{\cos(\theta(t))}{v(t)} & 0 < y(t) \\
  0 & \text{otherwise}
\end{cases} \\
\frac{d}{dt} v(t) &= \begin{cases} 
  -\sin(\theta(t)) - Rv(t)^2 & 0 < y(t) \\
  0 & \text{otherwise}
\end{cases} \\
\frac{d}{dt} x(t) &= \begin{cases} 
  v(t) \cos(\theta(t)) & 0 < y(t) \\
  0 & \text{otherwise}
\end{cases} \\
\frac{d}{dt} y(t) &= \begin{cases} 
  v(t) \sin(\theta(t)) & 0 < y(t) \\
  0 & \text{otherwise}
\end{cases}
\end{align*}
\right]
\]
\begin{verbatim}
R := 1;
stuff := [theta(t), v(t), x(t), y(t)], t = -1..20,
theta = -Pi..4*Pi, v = 0..3, x = -3..10, y = -2..3,
[seq([theta(0) = 0, v(0) = i, x(0) = 0, y(0) = 0], i = 1..3, 0.2)],
linecolor = [seq(COLOR(HUE, i), i = 0..1, .1)], stepsize = 0.05:
display( array( [DEplot(crashy, stuff, scene = [theta, v]),
DEplot(crashy, stuff, scene = [x, y]) ]));

R := 0.5;
stuff := [theta(t), v(t), x(t), y(t)], t = -1..20,
theta = -Pi..4*Pi, v = 0..3, x = -3..10, y = -2..3,
[seq([theta(0) = 0, v(0) = i, x(0) = 0, y(0) = 1], i = 1..3, 0.2)],
linecolor = [seq(COLOR(HUE, i), i = 0..1, .1)], stepsize = 0.05:
display( array( [DEplot(crashy, stuff, scene = [theta, v]),
DEplot(crashy, stuff, scene = [x, y]) ]));
\end{verbatim}