aside: how to do second derivative?

```
> diff(y(t),t);
```

$$\frac{d}{dt} y(t) \quad (1)$$

```
> diff(diff(y(t),t),t);
```

$$\frac{d^2}{dt^2} y(t) \quad (2)$$

```
> D(y)(t);
```

$$D(y)(t) \quad (3)$$

```
> D(D(sin));
```

$$-\sin \quad (4)$$

```
> diff(y(t),t,t,t,t);
```

$$\frac{d^4}{dt^4} y(t) \quad (5)$$

```
> diff(y(t),t$4);
```

$$\frac{d^4}{dt^4} y(t) \quad (6)$$

```
> t$10
```

Warning, inserted missing semicolon at end of statement

$$t, t, t, t, t, t, t, t, t, t \quad (7)$$

```
> diff(y(t),t$10);
```

$$\frac{d^{10}}{dt^{10}} y(t) \quad (8)$$

```
> with(DEtools):
  with(plots):
> phug:=[ D(theta)(t) = v(t) - cos(theta(t))/v(t),
        D(v)(t)     = -sin(theta(t)) -R*v(t)^2];
```

$$phug := \left[D(\theta)(t) = v(t) - \frac{\cos(\theta(t))}{v(t)}, D(v)(t) = -\sin(\theta(t)) - R\, v(t)^2 \right] \quad (9)$$

```
> xphug:=[ D(theta)(t) = v(t) - cos(theta(t))/v(t),
         D(v)(t)     = -sin(theta(t)) -R*v(t)^2,
         D(x)(t)     = v(t)*cos(theta(t)),
         D(y)(t)     = v(t)*sin(theta(t))];
```

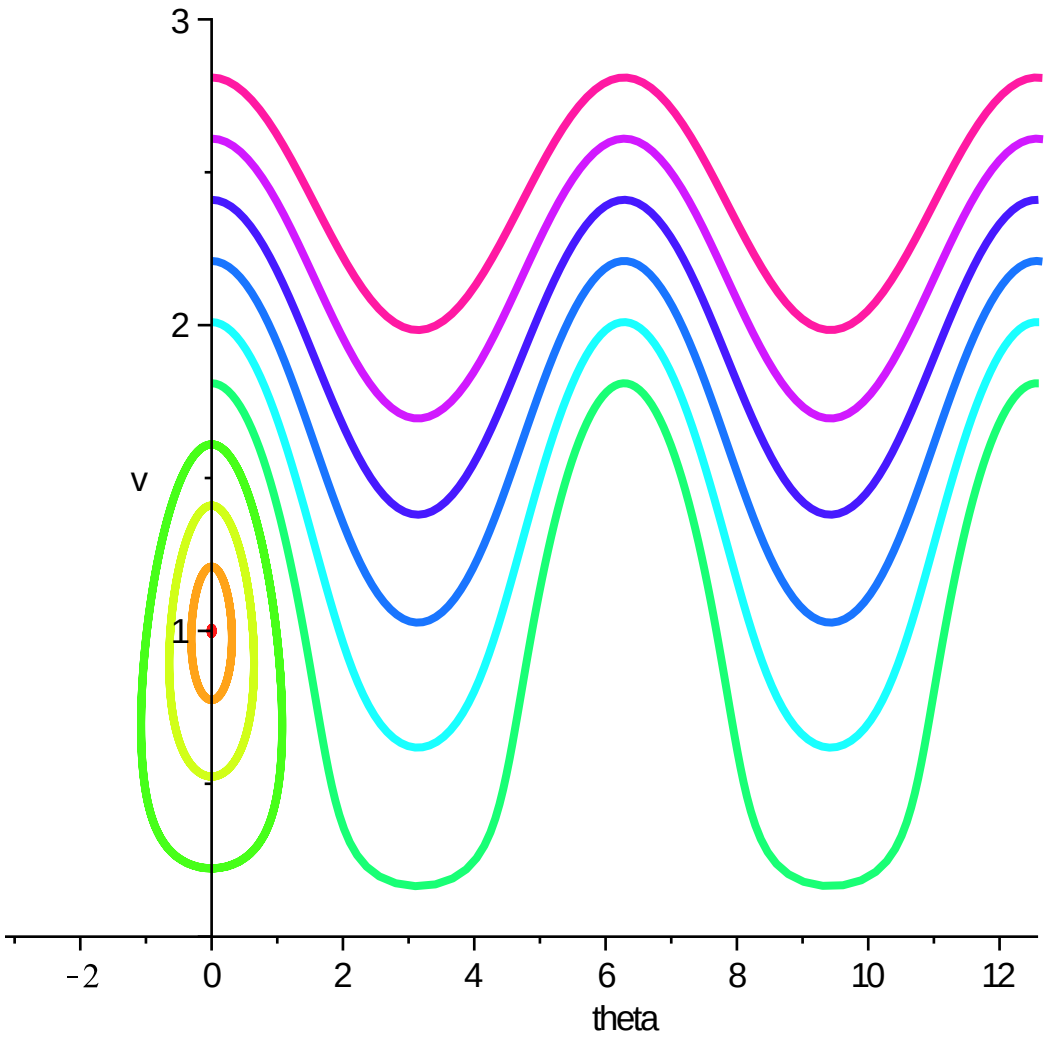
$$xphug := \left[D(\theta)(t) = v(t) - \frac{\cos(\theta(t))}{v(t)}, D(v)(t) = -\sin(\theta(t)) - R\, v(t)^2, D(x)(t) \right. \quad (10)$$

$$\left. = v(t)\cos(\theta(t)), D(y)(t) = v(t)\sin(\theta(t)) \right]$$

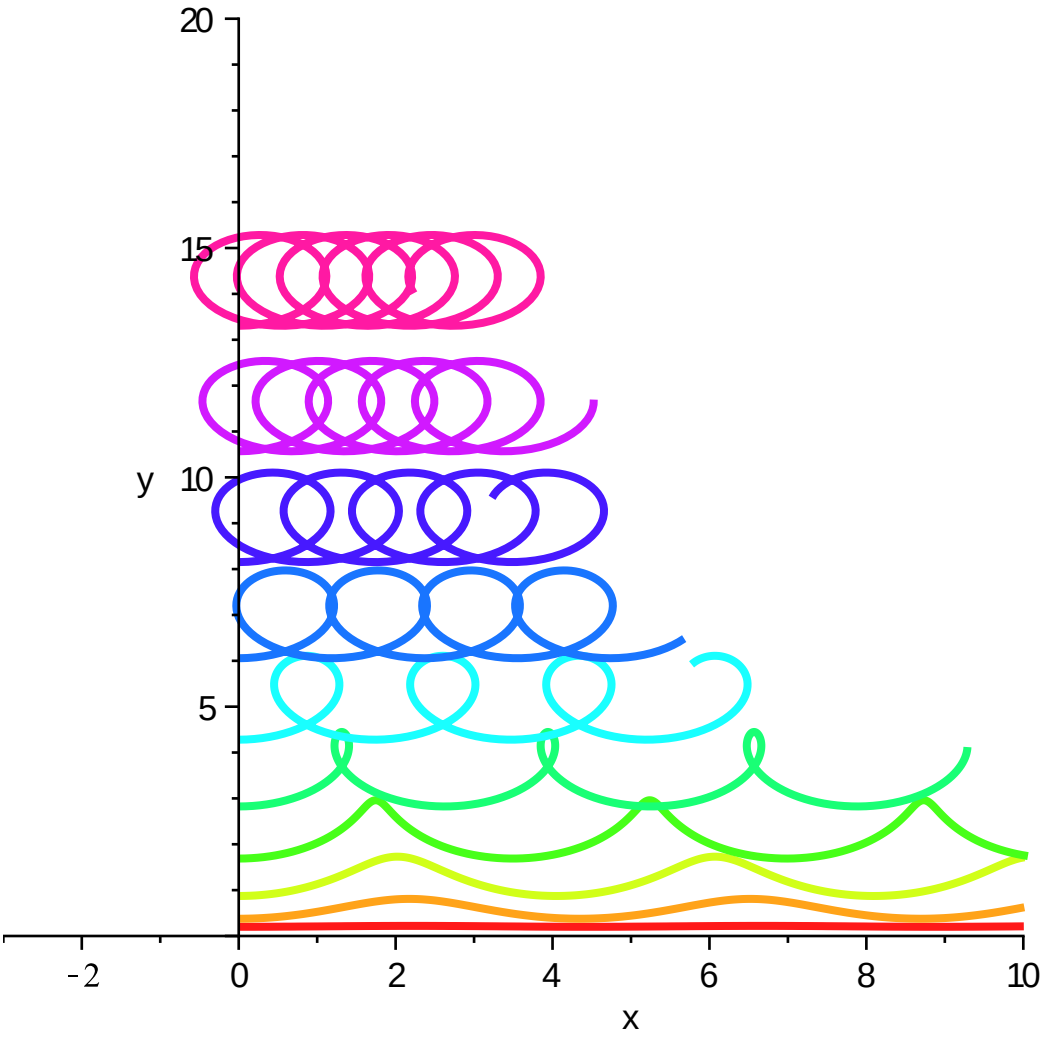
```
> stuff:=[theta(t), v(t), x(t), y(t)], t=0..15,
   theta=-Pi..4*Pi, v=0..3, x=-3..10, y=0..20,
   [seq([theta(0)=0, v(0)=i, x(0)=0, y(0)=4*(i-1)^2+.2],i=1.01..3,
  0.2)],
   linecolor=[seq(COLOR(HUE,i),i=0..1,.1)], stepsize=0.05:
```

```
> R:=0;
  DEplot(xphug, stuff, scene=[theta,v]);
```

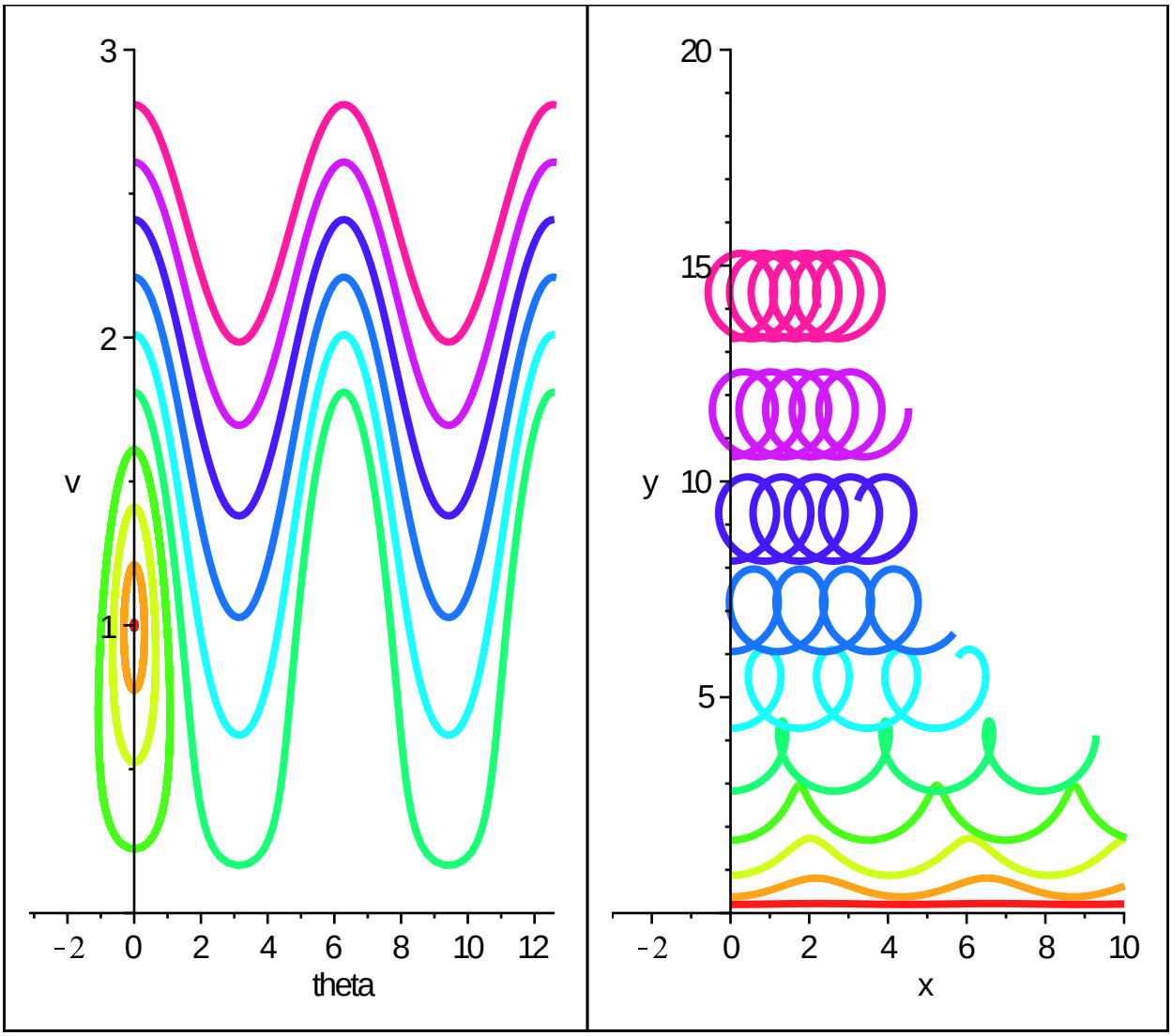
$$R := 0$$



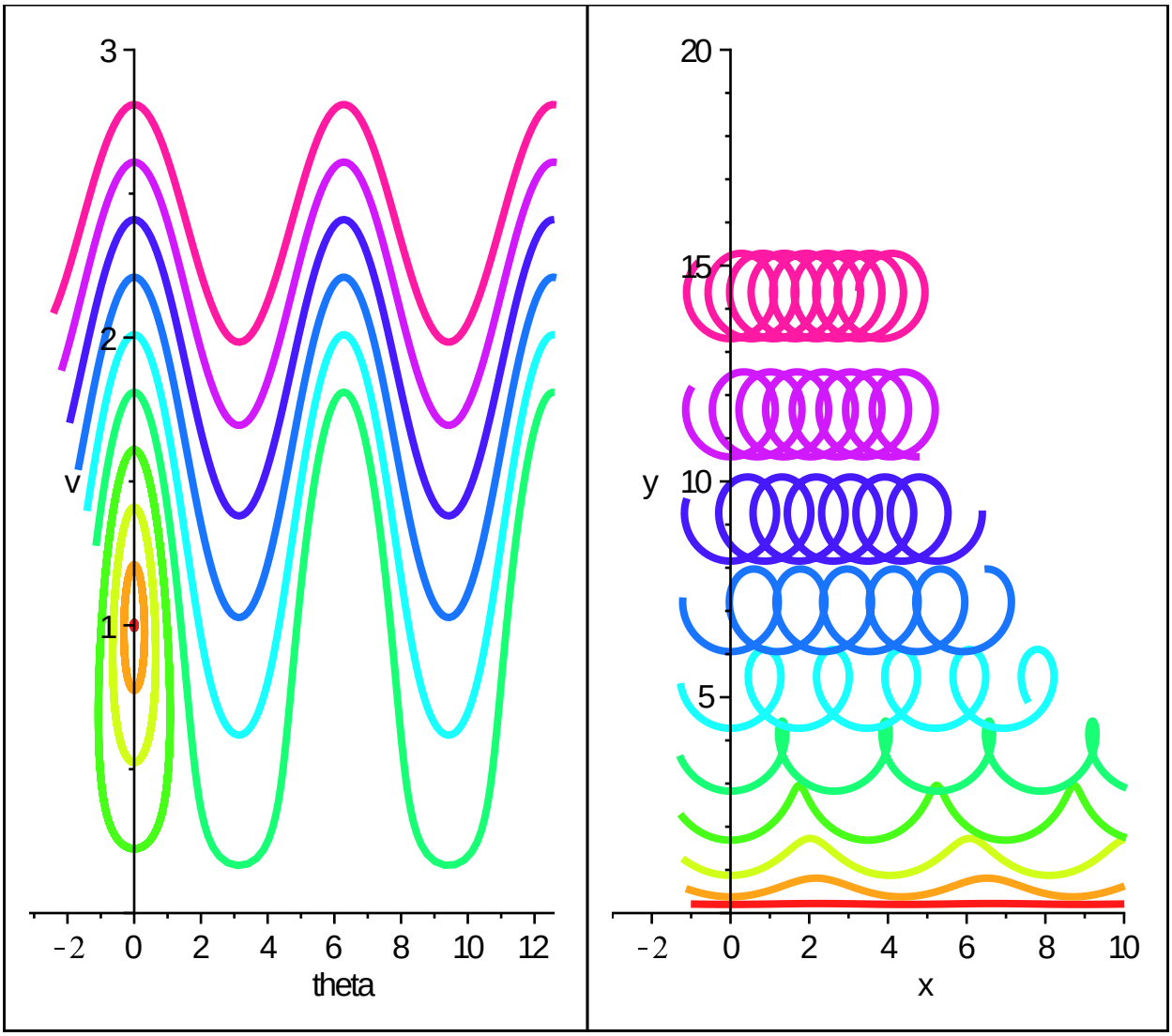

```
> DEplot(xphug, stuff, scene=[x,y]);
```



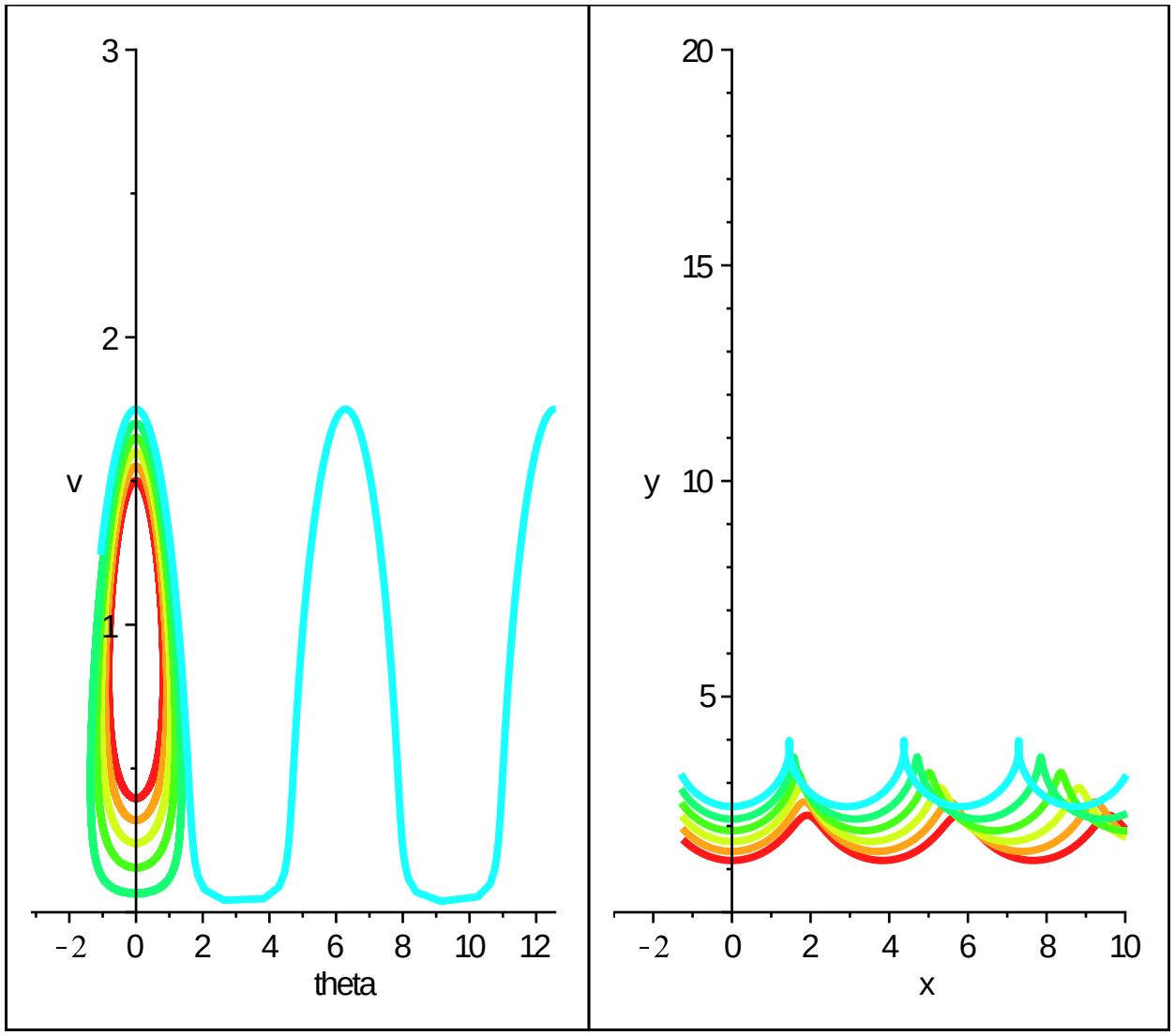

```
> display( array( [ DEplot(xphug, stuff, scene=[theta,v]),
                    DEplot(xphug, stuff, scene=[x,y]) ]));
```




```
> stuff:=[theta(t), v(t), x(t), y(t)], t=-1..20,
   theta=-Pi..4*Pi, v=0..3, x=-3..10, y=0..20,
   [seq([theta(0)=0, v(0)=i, x(0)=0, y(0)=4*(i-1)^2+.2],i=1.01..3,
  0.2)],
   linecolor=[seq(COLOR(HUE,i),i=0..1,.1)], stepsize=0.05:
  display( array( [ DEplot(xphug, stuff, scene=[theta,v]),
                    DEplot(xphug, stuff, scene=[x,y]) ]));
```




```
> stuff:=[theta(t), v(t), x(t), y(t)], t=-1..20,
   theta=-Pi..4*Pi, v=0..3, x=-3..10, y=0..20,
   [seq([theta(0)=0, v(0)=i, x(0)=0, y(0)=4*(i-1)^2+.2],i=1.5.
  .1.75,0.05)],
   linecolor=[seq(COLOR(HUE,i),i=0..1,.1)], stepsize=0.05:
  display( array( [ DEplot(xphug, stuff, scene=[theta,v]),
                    DEplot(xphug, stuff, scene=[x,y]) ]));
```

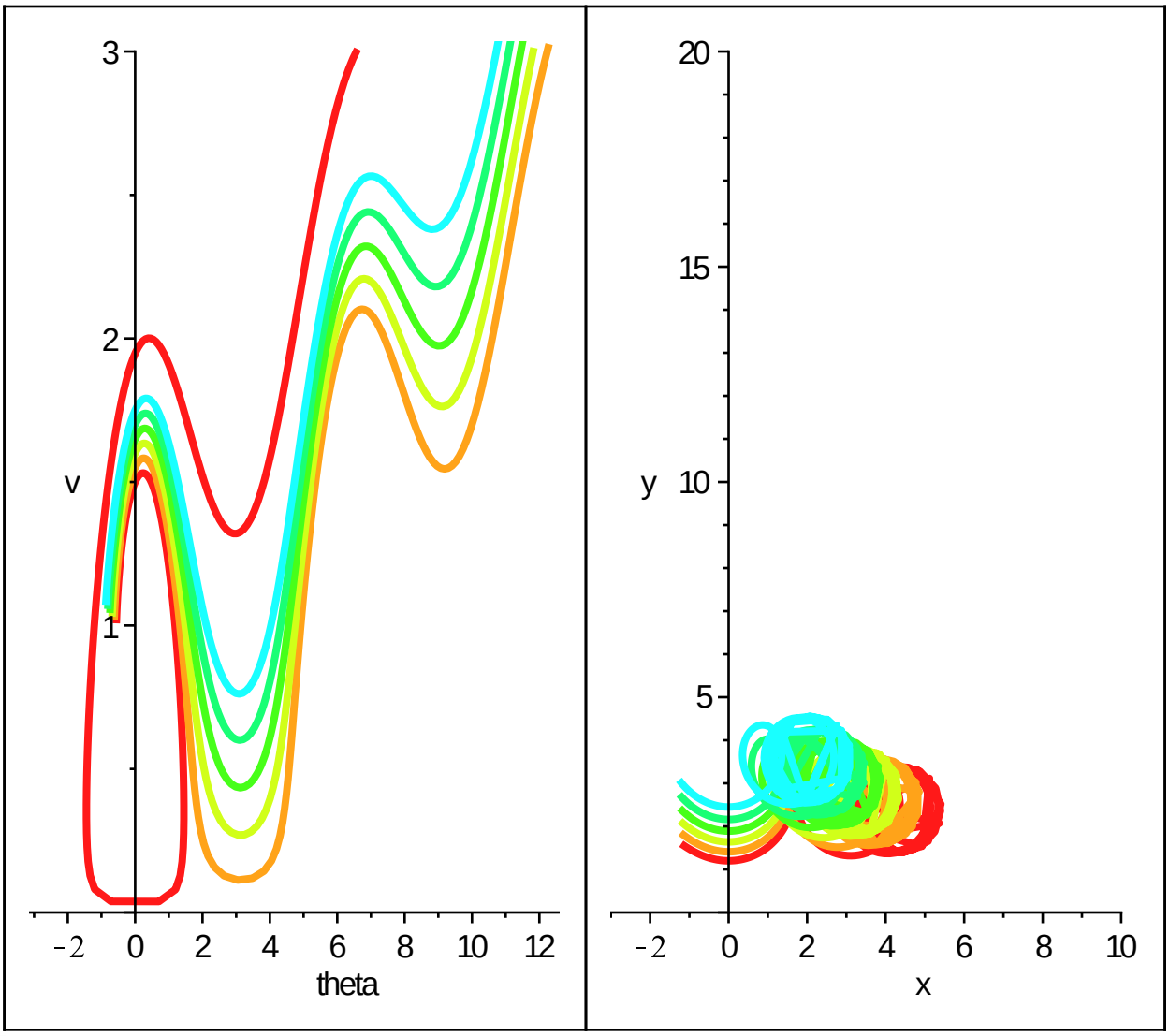



```
> R:=-0.1;
  stuff:=[theta(t), v(t), x(t), y(t)], t=-1..20,
    theta=-Pi..4*Pi, v=0..3, x=-3..10, y=0..20,
    [seq([theta(0)=0, v(0)=i, x(0)=0, y(0)=4*(i-1)^2+.2],i=1.5.
  .1.75,0.05)],
    linecolor=[seq(COLOR(HUE,i),i=0..1,.1)], stepsize=0.05:
  display( array( [ DEplot(xphug, stuff, scene=[theta,v]),
                    DEplot(xphug, stuff, scene=[x,y]) ]));
```

$$R := -0.1$$

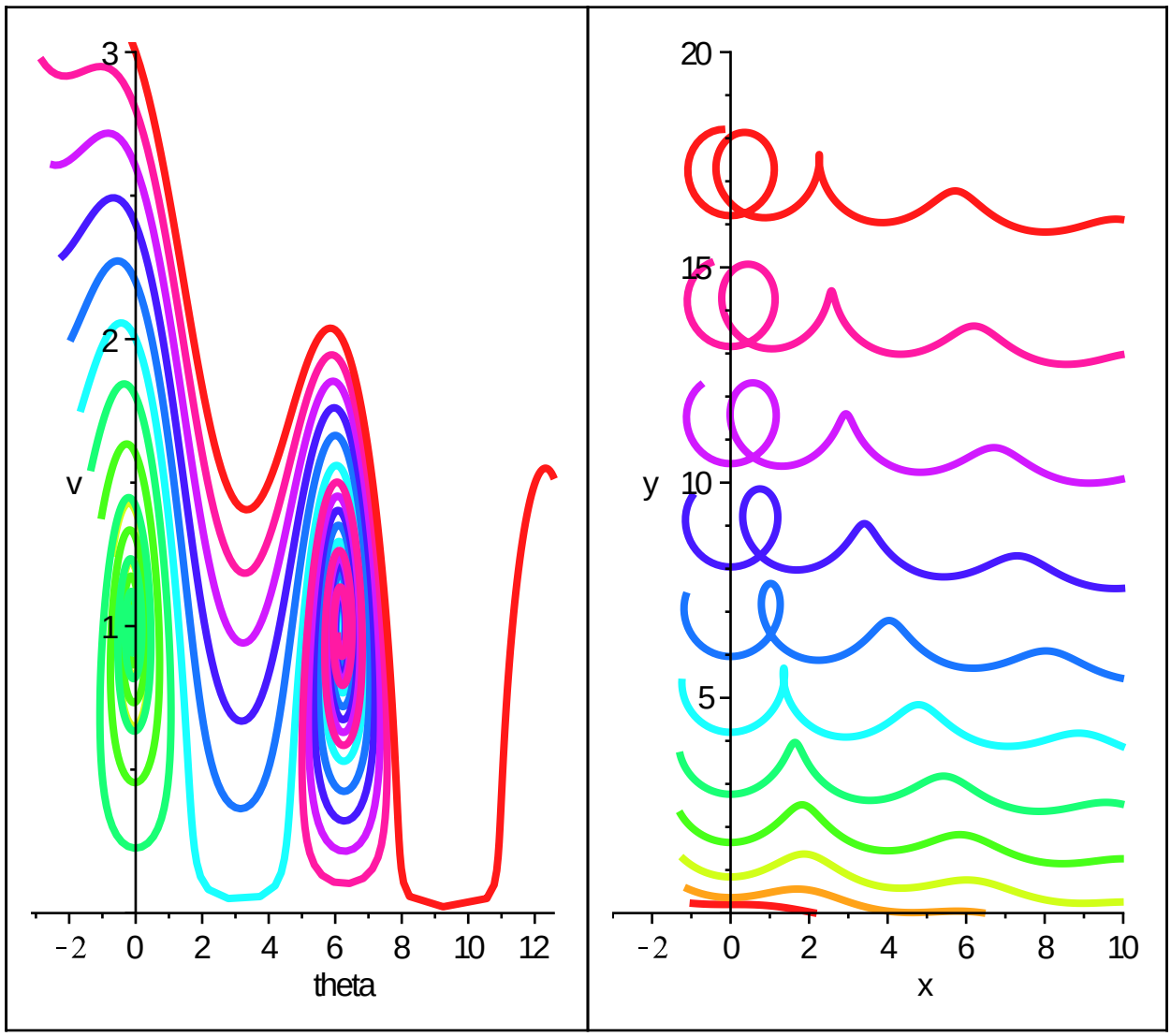
Warning, plot may be incomplete, the following errors(s) were issued:
cannot evaluate the solution further right of 11.748366, probably a singularity
Warning, plot may be incomplete, the following errors(s) were issued:
cannot evaluate the solution further right of 11.140667, probably a singularity
Warning, plot may be incomplete, the following errors(s) were issued:
cannot evaluate the solution further right of 10.557935, probably a singularity

```
Warning, plot may be incomplete, the following errors(s) were
issued:
   cannot evaluate the solution further right of 9.9771508,
probably a singularity
Warning, plot may be incomplete, the following errors(s) were
issued:
   cannot evaluate the solution further right of 9.4067059,
probably a singularity
Warning, plot may be incomplete, the following errors(s) were
issued:
   cannot evaluate the solution further right of 8.8653095,
probably a singularity
```



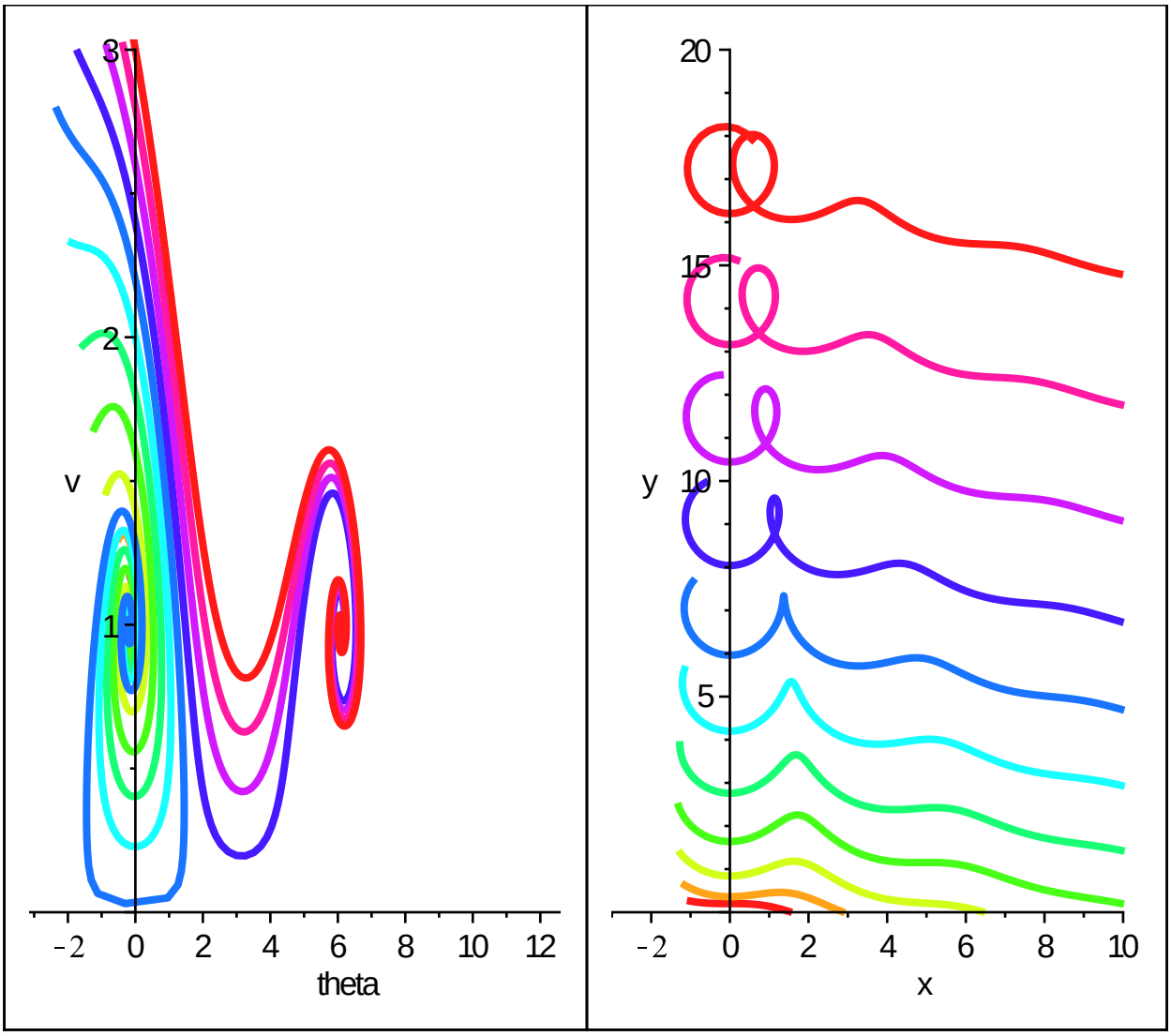

```
> R:=0.1;
  stuff:=[theta(t), v(t), x(t), y(t)], t=-1..20,
    theta=-Pi..4*Pi, v=0..3, x=-3..10, y=0..20,
    [seq([theta(0)=0, v(0)=i, x(0)=0, y(0)=4*(i-1)^2+.2],i=1..3,
  0.2)],
    linecolor=[seq(COLOR(HUE,i),i=0..1,.1)], stepsize=0.05:
  display( array( [ DEplot(xphug, stuff, scene=[theta,v]),
                    DEplot(xphug, stuff, scene=[x,y]) ]));
```

$R := 0.1$




```
> R:=0.2;
  display( array( [ DEplot(xphug, stuff, scene=[theta,v]),
                    DEplot(xphug, stuff, scene=[x,y]) ]));
```

$R := 0.2$

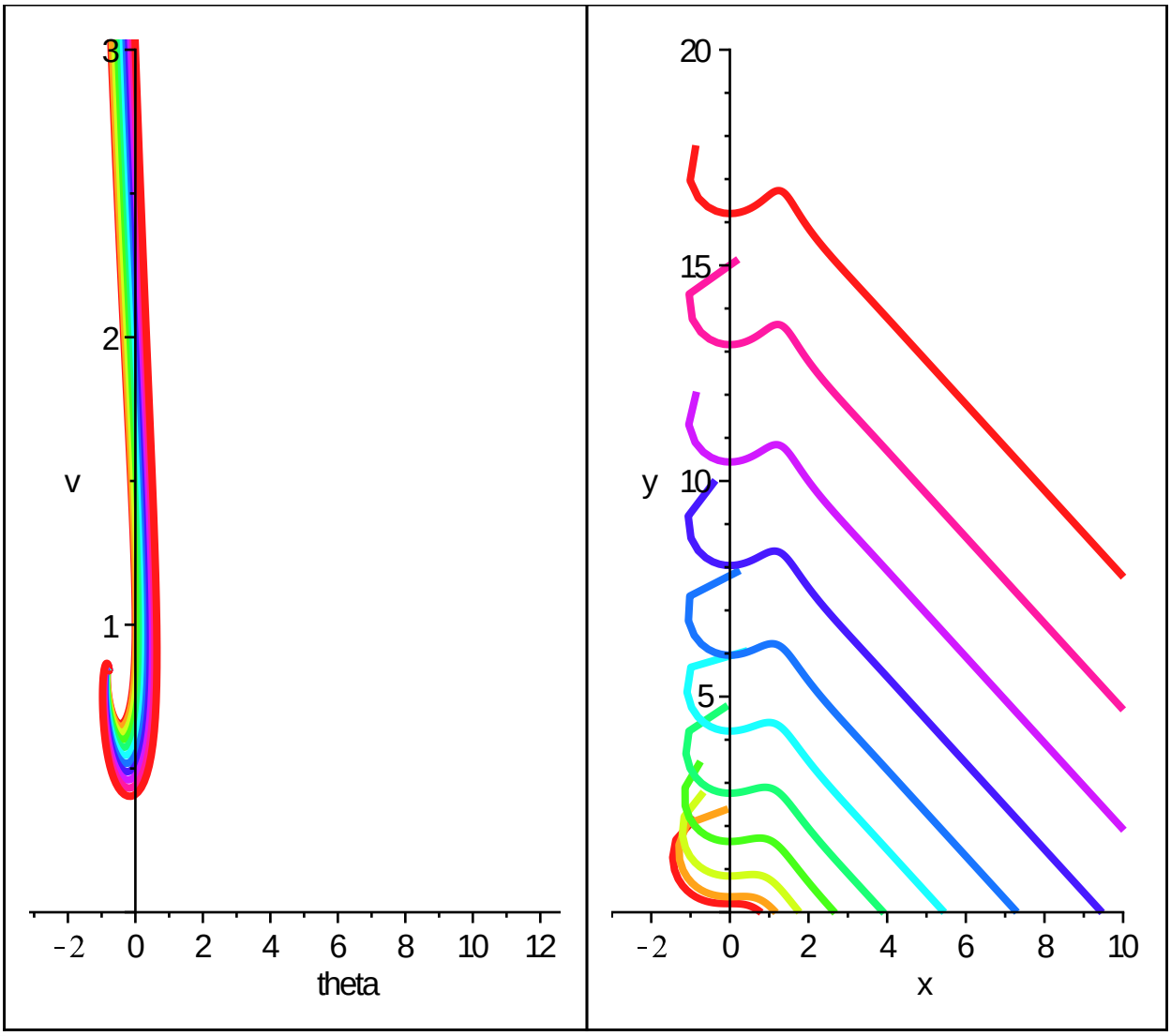



```
> R:=1;
  display( array( [ DEplot(xphug, stuff, scene=[theta,v]),
                    DEplot(xphug, stuff, scene=[x,y]) ]));
```

$$R := 1$$

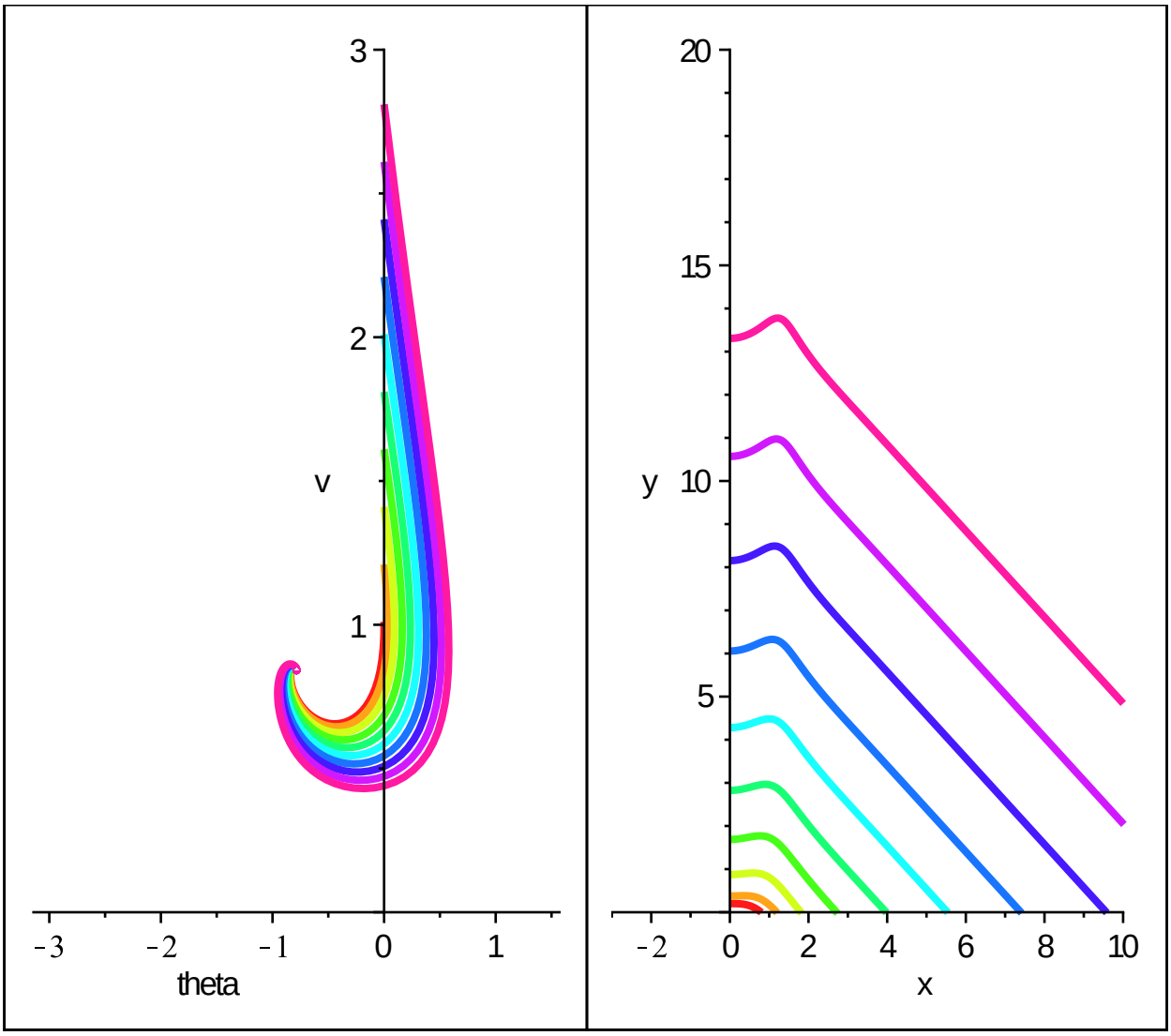
```
Warning, plot may be incomplete, the following errors(s) were
issued:
   cannot evaluate the solution further left of -.87237356,
probably a singularity
Warning, plot may be incomplete, the following errors(s) were
issued:
   cannot evaluate the solution further left of -.74284560,
probably a singularity
Warning, plot may be incomplete, the following errors(s) were
issued:
   cannot evaluate the solution further left of -.64565873,
probably a singularity
Warning, plot may be incomplete, the following errors(s) were
issued:
   cannot evaluate the solution further left of -.57072882,
probably a singularity
Warning, plot may be incomplete, the following errors(s) were
```

issued:
cannot evaluate the solution further left of -.51138114, probably a singularity
Warning, plot may be incomplete, the following errors(s) were issued:
cannot evaluate the solution further left of -.46326338, probably a singularity
Warning, plot may be incomplete, the following errors(s) were issued:
cannot evaluate the solution further left of -.42347492, probably a singularity
Warning, plot may be incomplete, the following errors(s) were issued:
cannot evaluate the solution further left of -.39002515, probably a singularity
Warning, plot may be incomplete, the following errors(s) were issued:
cannot evaluate the solution further left of -.36150801, probably a singularity
Warning, plot may be incomplete, the following errors(s) were issued:
cannot evaluate the solution further left of -.33690396, probably a singularity



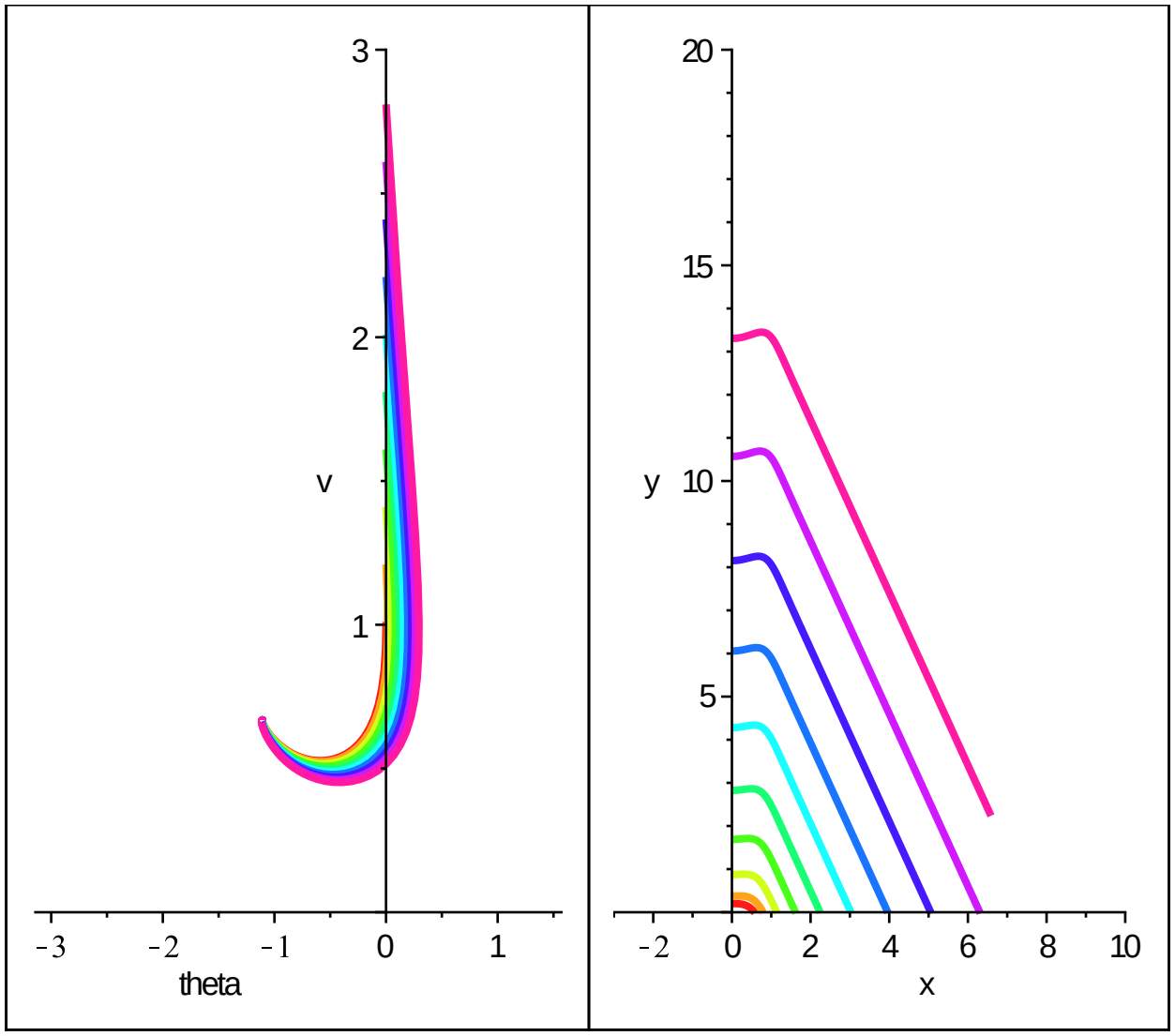

```
> stuff:=[theta(t), v(t), x(t), y(t)], t=0..20,
   theta=-Pi..Pi/2, v=0..3, x=-3..10, y=0..20,
   [seq([theta(0)=0, v(0)=i, x(0)=0, y(0)=4*(i-1)^2+.2],i=1.01..3,
  0.2)],
   linecolor=[seq(COLOR(HUE,i),i=0..1,.1)], stepsize=0.05:
> R:=1;
  display( array( [ DEplot(xphug, stuff, scene=[theta,v]),
                    DEplot(xphug, stuff, scene=[x,y]) ]));
```

$$R := 1$$




```
> R:=2;
  display( array( [ DEplot(xphug, stuff, scene=[theta,v]),
                    DEplot(xphug, stuff, scene=[x,y]) ]));
```

$$R := 2$$




```
> R:='R';
```

$$R := R \tag{11}$$

```
> phug;
```

$$\left[\mathrm{D}(\theta)(t) = v(t) - \frac{\cos(\theta(t))}{v(t)}, \mathrm{D}(v)(t) = -\sin(\theta(t)) - R\, v(t)^2 \right] \tag{12}$$

```
> solve( { -sin(theta)-R*v^2 = 0 , v-cos(theta)/v=0}, {theta,v});
```

$$\{v = RootOf(-RootOf(-1 + (R^2 + 1)\_Z^2) + \_Z^2), \theta = \arctan(-RootOf(-1 + (R^2 + 1)\_Z^2)\, R, RootOf(-1 + (R^2 + 1)\_Z^2))\} \tag{13}$$

```
> convert(%,radical);
```

$$\left\{ v = \left(\frac{1}{R^2+1} \right)^{1/4}, \theta = \arctan\left(-\sqrt{\frac{1}{R^2+1}}\, R, \sqrt{\frac{1}{R^2+1}} \right) \right\} \tag{14}$$