## MAT331 Exercises, Fall 2019

36. (*expires 12/4*) The difficulty of breaking a cipher can be increased by inserting some random characters (or noise) into a known part of the plain text in such a way that it will interact with the actual text (sometimes this insertion of randomness is called "salting the plaintext").

As an example of this, recall that we discussed (on Nov. 21) that if our character set came from an alphabet of length *n*, we could represent blocks of *k*-characters as numbers base  $n^k$  (for example, in the 53-character alphabet consisting of a space, upper-case letters A-Z, and lower-case letters a-z, the word Hi is represented by  $8 + 35 \cdot 53 = 1863$  if we use 2-character blocks).

If we agree that the first character of each block will be randomly chosen (and just removed after we decrypt), breaking the encryption becomes much, much harder. Indeed, the difficulty decreases by a factor of the number of possible salt characters.

Modify the encryption scheme in Affine from Crypto.mw so that it can encrypt and decrypt including salt as the first character of each block. You may either make a new procedure SaltedAffine or modify Affine to accept an optional parameter to insert (and remove) the salt, as you see fit.

As an explicit example, let us encode the string Zombie Apocalypse using the 53-character alphabet and 3-character blocks (where the first character of each block is randomly chosen from the Alphabet). In this example, the given string corresponds to the list of character codes

[26, 41, 39, 28, 35, 31, 0, 1, 42, 41, 29, 27, 38, 51, 42, 45, 31]

which, when grouped into pairs and with a random character added as salt, gives

[116551, 80721, 88936, 2842, 117404, 77428, 145275, 128676, 1652]

(your numbers may differ by up to 53 because of the salt); viewing these as 3-graphs including the salt, corresponds to DZoBmbBieg AIpovcaBlyspsIe. If we then encrypt this using the affine encryption  $x \mapsto 12347x + 56890 \mod 53^3$ , we get

[67005, 136439, 32930, 12092, 28729, 121189, 97219, 4118, 57985]

or MsWQdvQlKHPDCLJeGqQfhkXAChT (unless you used the same salt, your encryption will differ significantly, but both should decrypt just fine.)

If you were able to make this work, you should be able to decrypt the string<sup>1</sup>

MWLiuWVckaMOpfHuWPgGuWlyQovqkwBwWV nLZYhrySYihTUSFqFJuGxEtvxCWNxPxstkPwkkxo

which was encrypted using salted pairs (that is, 3-character blocks including salt) from the 53character alphabet above, and applying the affine mapping  $x \mapsto 47x + 31415 \mod 53^3$ . Decrypt the phrase.

<sup>&</sup>lt;sup>1</sup>also available as zombiecrypt.txt so you don't have to worry about typing errors.