2019-10-31

Happy Halloween

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> punkin(.5)
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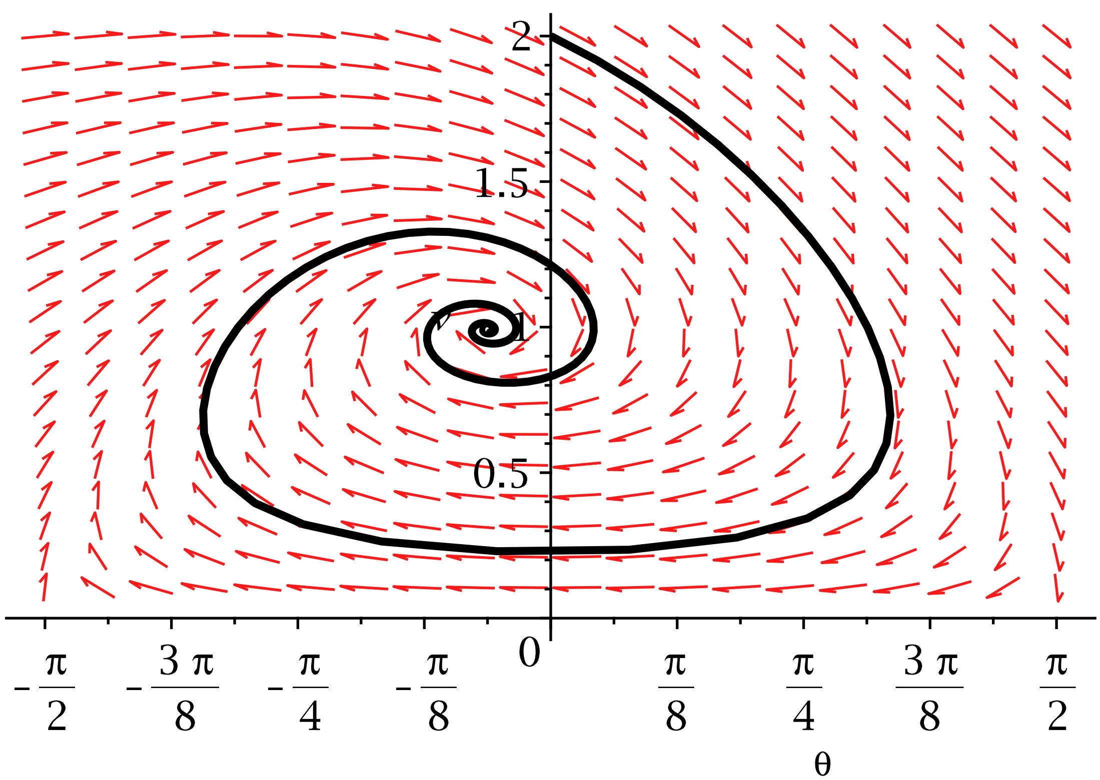
So, more glidery stuff.

> $with(DEtools):$

> $phug(R) := \left[diff(\theta(t), t) = v(t) - \frac{\cos(\theta(t))}{v(t)}, diff(v(t), t) = -\sin(\theta(t)) - R \cdot v(t)^2 \right]:$

> $DEplot\left(phug(.2), [\theta, v], t = 0..30, [[\theta(0) = 0, v(0) = 2]], \theta = -\frac{Pi}{2}..\frac{Pi}{2}, v = 0..2, linecolor = black, tickmarks = [piticks, default], stepsize = .1 \right)$

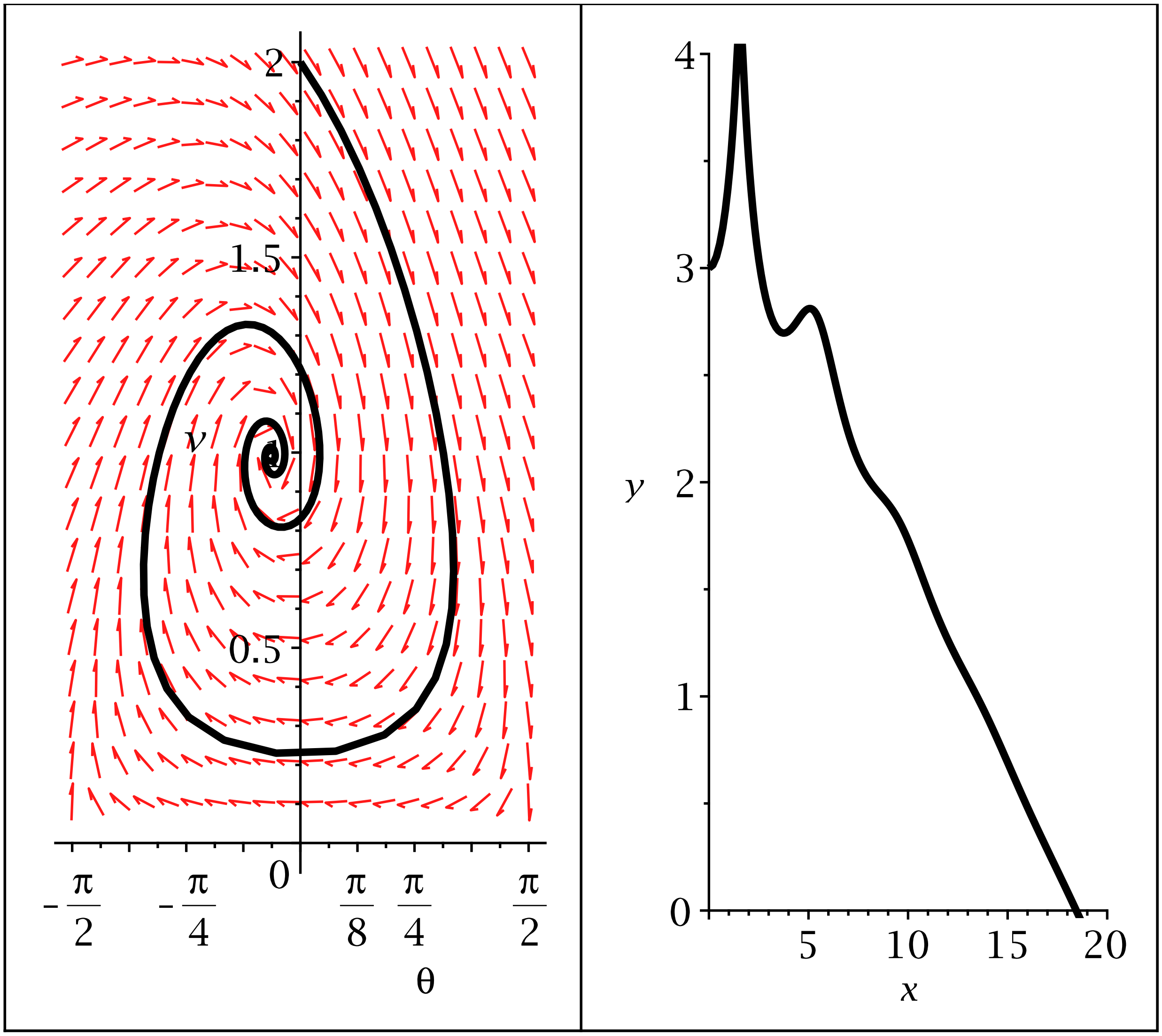


Given initial angle, velocity, height.... how far does the glider go before it crashes into the ground?

Step 1: Add x, y to equations.

> $xphug(R) := [op(phug(R)),\ diff(x(t), t) = v(t)\cdot\cos(\theta(t)), diff(y(t), t) = v(t)\cdot\sin(\theta(t))]:$

> $plots[display]\Big(\Big\langle DEplot\Big(phug(.2), [\theta, v], t = 0..30, [[\theta(0) = 0, v(0) = 2]], \theta = -\frac{Pi}{2}..\frac{Pi}{2}, v = 0..2, linecolor = black, tickmarks = [piticks, default], stepsize = .1\Big) \Big| DEplot\Big(xphug(.2), [\theta, v, x, y], t = 0..30, [[\theta(0) = 0, v(0) = 2, x(0) = 0, y(0) = 3]], \theta = -\frac{Pi}{2}..\frac{Pi}{2}, v = 0..2, x = 0..20, y = 0..4,\ scene = [x, y], obsrange = false, linecolor = black, stepsize = .1\Big)\Big\rangle\Big)$



> *plots*[*display*]$\Big(\Big\langle$*DEplot*$\Big($*phug*(.2), [theta, *v*], *t* = 0 ..30,

$\Big[$[theta(0) = 0, *v*(0) = 2], $\Big[$theta(0) = $-\frac{\text{Pi}}{4}$, *v*(0) = 2$\Big]\Big]$, theta = $-\frac{\text{Pi}}{2}$ .. $\frac{\text{Pi}}{2}$, *v* = 0 ..2,

linecolor = [*black*, *blue*], *tickmarks* = [*piticks*, *default*], *stepsize* = .1$\Big)$ $\Big|$

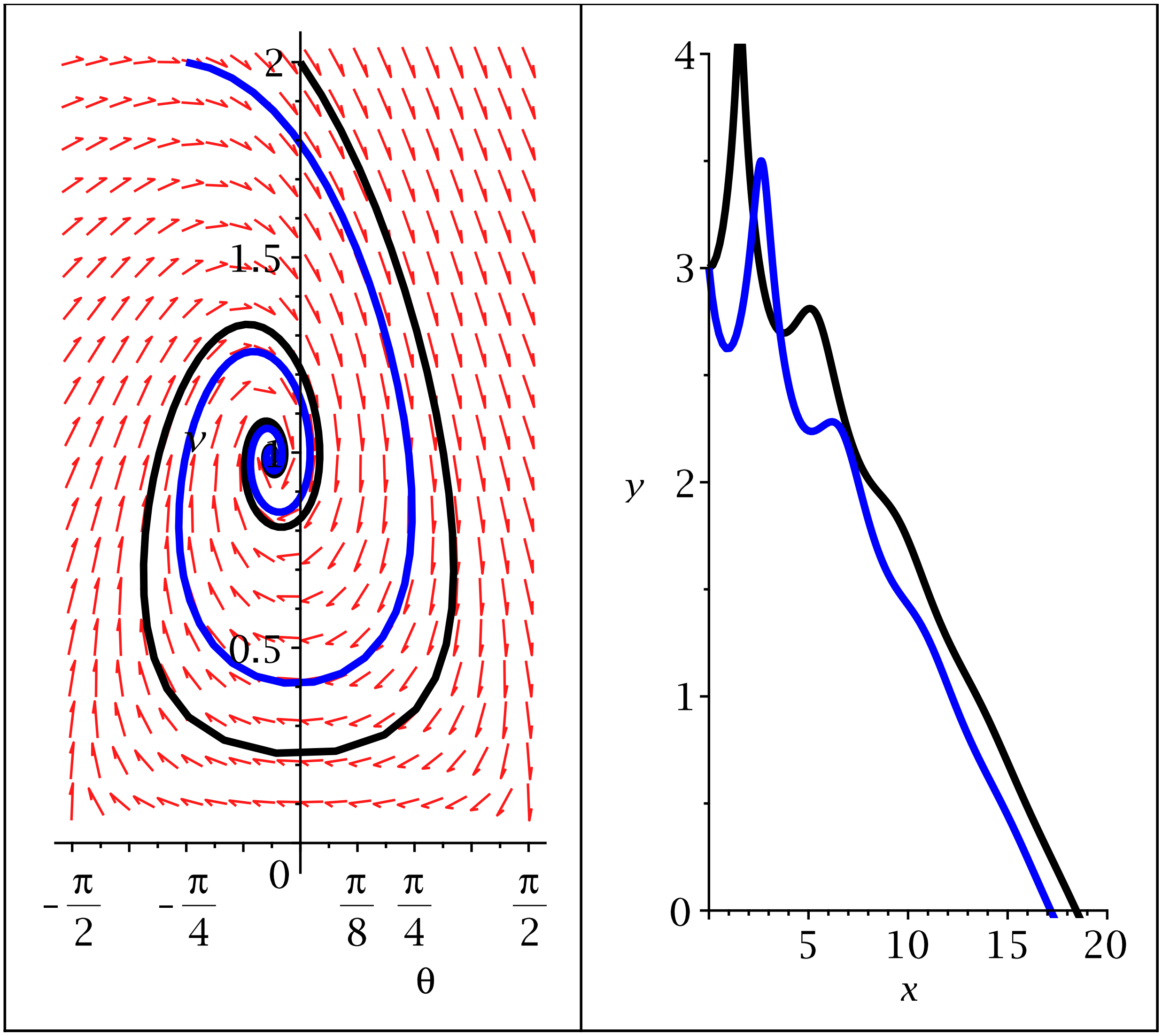
DEplot$\Big($*xphug*(.2), [theta, *v*, *x*, *y*], *t* = 0 ..30,

$\Big[$[theta(0) = 0, *v*(0) = 2, *x*(0) = 0, *y*(0) = 3], $\Big[$theta(0) = $-\frac{\text{Pi}}{4}$, *v*(0) = 2, *x*(0) = 0,

y(0) = 3$\Big]\Big]$,

theta = $-\frac{\text{Pi}}{2}$ .. $\frac{\text{Pi}}{2}$, *v* = 0 ..2, *x* = 0 ..20, *y* = 0 .. 4, *scene* = [*x*, *y*], *obsrange* = *false*,

linecolor = [*black*, *blue*], *stepsize* = .1$\Big)\Big\rangle\Big)$



What is the distance? Let's actually solve numerically

This can be done using dsolve with the numeric option which returns a procedure as output.

> $sol0 := dsolve(\{op(xphug(.2)), \theta(0) = 0, v(0) = 2, x(0) = 0, y(0) = 3\}, numeric, stepsize = .1)$

$$sol0 := \mathbf{proc}(x\_rkf45)\ \ldots\ \mathbf{end\ proc} \quad (1)$$

> $sol0(3)$

$$[t = 3., \theta(t) = -1.01170079409444, v(t) = 0.933293607714441, x(t) = 1.87261979030816, y(t) = 3.68812285099328] \quad (2)$$

> $sol0(20)$

$$[t = 20., \theta(t) = -0.196604734090982, v(t) = 0.987028563714960, x(t) = 18.3888929766207, y(t) = 0.0114994944851027] \quad (3)$$

> $sol0(20.1)$

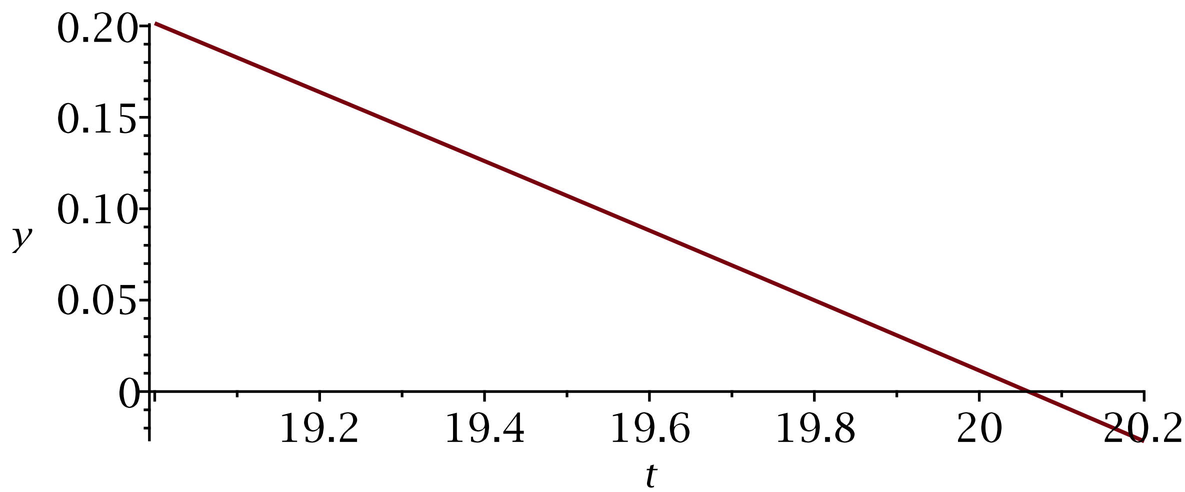
$$[t = 20.1000000000, \theta(t) = -0.1972508214, v(t) = 0.9871086505, x(t) \quad (4)$$

$= 18.4856915212,\ y(t) = -0.0078133698]$

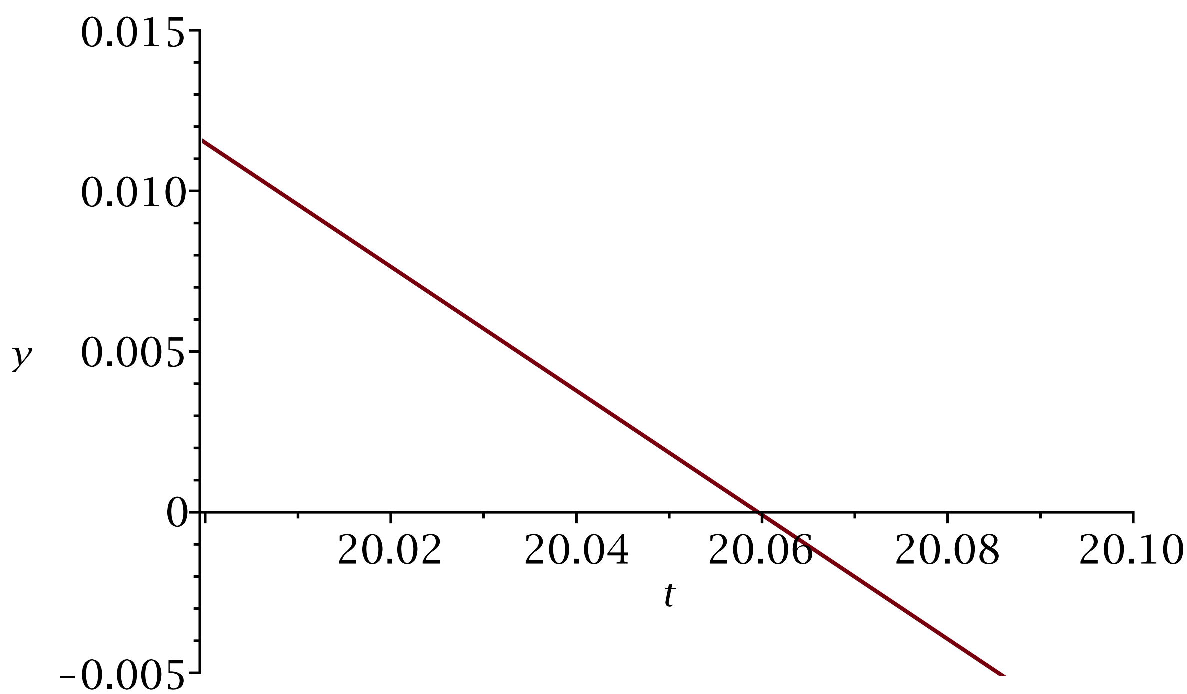
>

Instead of using DEplot as before, we could instead plot this particular numeric solution using odeplot from the plots library
Since there are many variables, we have to tell it which one we want.

> $plots[odeplot](sol0,\ [t,\ y(t)],\ t = 19..20.2)$



> $zoom(\%,\ 20..20.1,\ -.005..0.015)$



So it is pretty clear that the glider hits the ground at about t=20.06.

> $sol0(20.059);\ sol0(20.06)$

$[t = 20.0590000000,\ \theta(t) = -0.1969894633,\ v(t) = 0.9870685507,\ x(t)$
$= 18.4460046531,\ y(t) = 0.0001126554]$

$$[t = 20.0600000000, \theta(t) = -0.1969959010, v(t) = 0.9870694107, x(t) = 18.4469726318, y(t) = -0.0000805348] \quad (5)$$

And the distance is between 18.446 and 18.447

But wait! Planes don't fly underground!

Could we change the equations to reflect this fact?

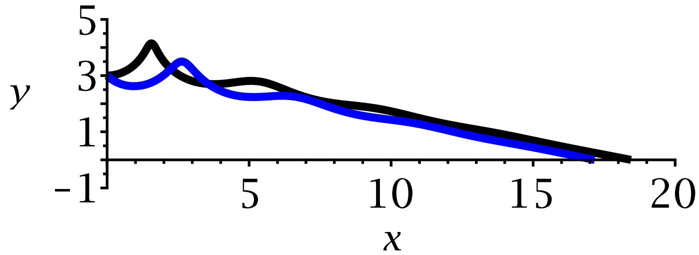
> $xphug(R)$

$$\left[\frac{d}{dt}\theta(t) = v(t) - \frac{\cos(\theta(t))}{v(t)}, \frac{d}{dt}v(t) = -\sin(\theta(t)) - R\,v(t)^2, \frac{d}{dt}x(t) = v(t)\cos(\theta(t)), \frac{d}{dt}y(t) = v(t)\sin(\theta(t))\right] \quad (6)$$

> $fixphug(R) := \left[\frac{d}{dt}\theta(t) = piecewise\left(y(t) > 0, v(t) - \frac{\cos(\theta(t))}{v(t)}, 0\right), \frac{d}{dt}v(t) = piecewise(y(t) > 0, -\sin(\theta(t)) - R\,v(t)^2, 0), \frac{d}{dt}x(t) = piecewise(y(t) > 0, v(t)\cos(\theta(t)), 0), \frac{d}{dt}y(t) = piecewise(y(t) > 0, v(t)\sin(\theta(t)), 0)\right]$

$$fixphug := R \mapsto \left[\frac{d}{dt}\theta(t) = \begin{cases} v(t) - \frac{\cos(\theta(t))}{v(t)} & 0 < y(t) \\ 0 & otherwise \end{cases}, \frac{d}{dt}v(t) = \begin{cases} -\sin(\theta(t)) - R\,v(t)^2 & 0 < y(t) \\ 0 & otherwise \end{cases}, \frac{d}{dt}x(t) = \begin{cases} v(t)\cos(\theta(t)) & 0 < y(t) \\ 0 & otherwise \end{cases}, \frac{d}{dt}y(t) = \begin{cases} v(t)\sin(\theta(t)) & 0 < y(t) \\ 0 & otherwise \end{cases}\right] \quad (7)$$

> $DEplot\Big(fixphug(.2), [\mathrm{theta}(t), v(t), x(t), y(t)], t = 0..25, \Big[[\mathrm{theta}(0) = 0, v(0) = 2, x(0) = 0, y(0) = 3], \Big[\mathrm{theta}(0) = -\frac{\mathrm{Pi}}{4}, v(0) = 2, x(0) = 0, y(0) = 3\Big]\Big], \mathrm{theta} = -\frac{\mathrm{Pi}}{2}..\frac{\mathrm{Pi}}{2}, v = 0..2, x = 0..20, y = -1..5, \; scene = [x, y], obsrange = false, linecolor = [black, blue], stepsize = .1, scaling = constrained\Big)$



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Aha! now it stops going when it hits the ground!

Next time, we'll finish this off.