2019-10-29 Let's deal with stalling.

$> R := 'R':$

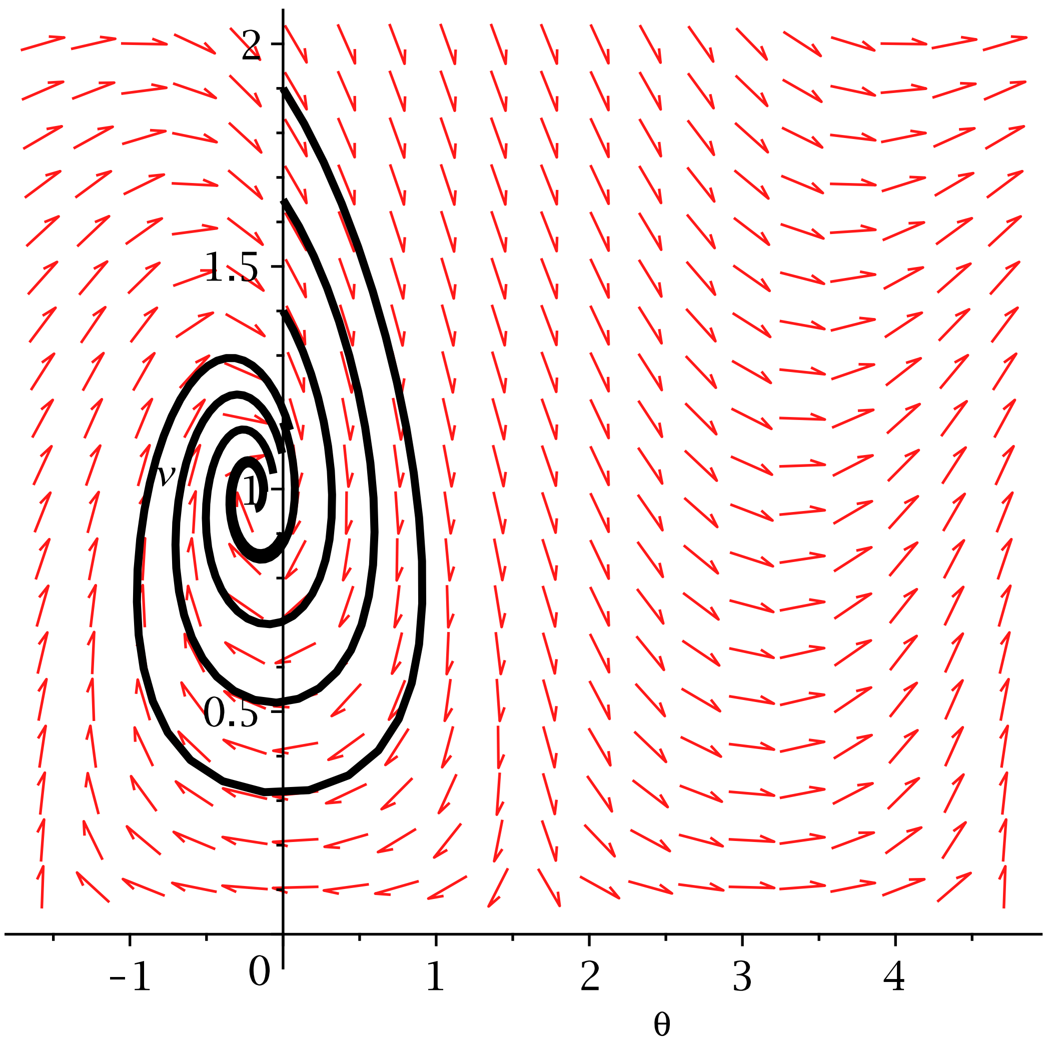
$$phug := \left[diff(\theta(t), t) = v(t) - \frac{\cos(\theta(t))}{v(t)}, diff(v(t), t) = -\sin(\theta(t)) - R \cdot v(t)^2 \right]:$$

$> with(DEtools):$

$> R := .2;$

$$DEplot\left(phug, [\theta, v], t = 0..5, [seq([v(0) = j, \theta(0) = 0], j = .9..3.0, .25)], \theta = -\frac{Pi}{2} .. \frac{3 \cdot Pi}{2}, v = 0..2, linecolor = black \right);$$

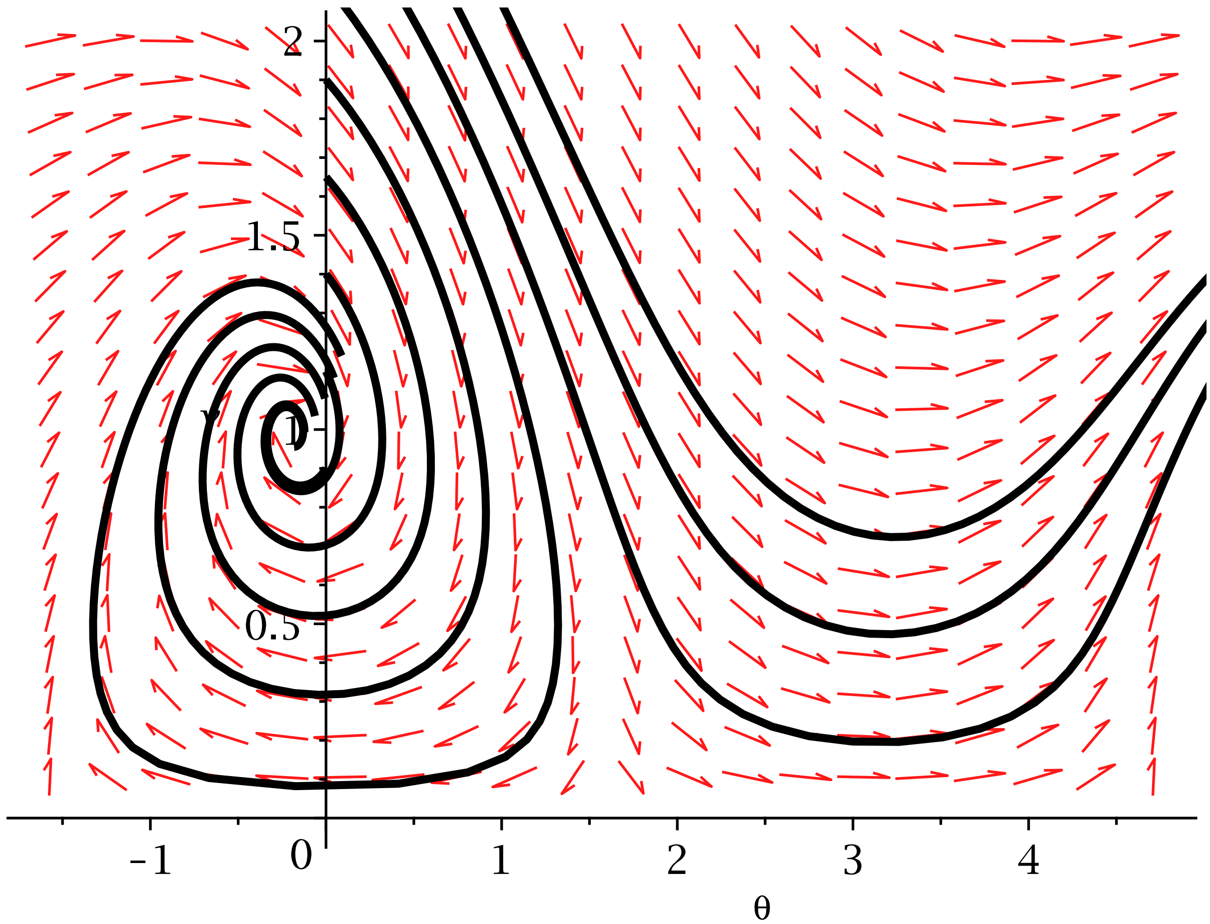
$$R := 0.2$$



$> R := .2;$

$$DEplot\left(phug, [\theta, v], t = 0..5, [seq([v(0) = j, \theta(0) = 0], j = .9..3.0, .25)], \theta = -\frac{Pi}{2} .. \frac{3 \cdot Pi}{2}, v = 0..2, linecolor = black, obsrange = false, stepsize = .05 \right);$$

$$R := 0.2$$



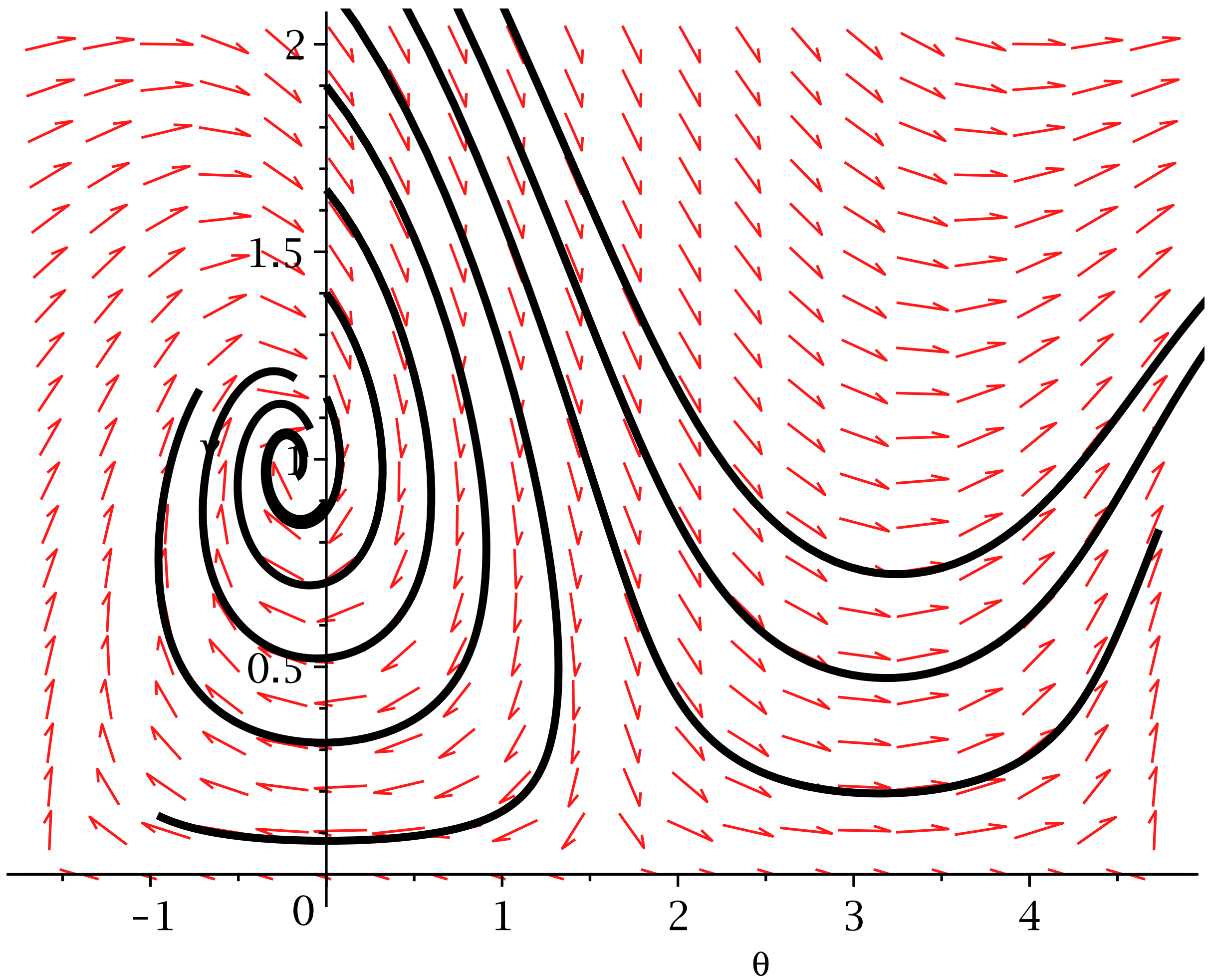
What does v=0, theta=Pi/2 mean for these equations?

Want to extend to v=0. Trick is to multiply both factors by v(t).

> $R := 'R'$:
$vphug := [\ diff(\text{theta}(t), t) = v(t)^2 - \cos(\text{theta}(t)), diff(v(t), t) = v(t) \cdot (-\sin(\text{theta}(t)) - R \cdot v(t)^2)\ ]$:

> $R := .2$;
$DEplot\left(\ vphug, [\text{theta}, v], t = 0..5, [seq([v(0) = j, \text{theta}(0) = 0], j = .9..3.0, .25)], \text{theta} = -\frac{\text{Pi}}{2}..\frac{3 \cdot \text{Pi}}{2}, v = 0..2, linecolor = black, obsrange = false, stepsize = .05\right)$;

$$R := 0.2$$



> $eval\left(map(rhs, vphug), \left\{v(t) = 0, \mathrm{theta}(t) = \frac{\mathrm{Pi}}{2}\right\}\right)$

$$[0, -0.] \quad (1)$$

Have a fixed point at v=0, theta=Pi/2 (also at -Pi/2). It should be a saddle. Let's check.

> $map(rhs, vphug)$

$$\left[v(t)^2 - \cos(\theta(t)), v(t)\left(-\sin(\theta(t)) - 0.2\, v(t)^2\right)\right] \quad (2)$$

> $subs(\{v(t) = v, \mathrm{theta}(t) = \mathrm{theta}\}, \%)$

$$\left[v^2 - \cos(\theta), v\left(-\sin(\theta) - 0.2\, v^2\right)\right] \quad (3)$$

> $F := unapply(\%, (\mathrm{theta}, v))$

$$F := (\theta, v) \mapsto \left[v^2 - \cos(\theta), v\left(-\sin(\theta) - 0.2\, v^2\right)\right] \quad (4)$$

> $solve(F(\mathrm{theta}, v))$

$$\{\theta = 1.570796327, v = 0.\}, \{\theta = -0.1973955598, v = 0.9902427357\}, \{\theta = -0.1973955598, v = -0.9902427357\}, \{\theta = 2.944197094, v = 0.9902427357\, \mathrm{I}\}, \{\theta = 2.944197094, v = -0.9902427357\, \mathrm{I}\} \quad (5)$$

> $with(LinearAlgebra):$
> $with(VectorCalculus):$

> *Jack* := *Jacobian*(*F*(theta, *v*), [theta, *v*])

$$Jack := \begin{bmatrix} \sin(\theta) & 2\,v \\ -v\cos(\theta) & -\sin(\theta) - 0.6\,v^2 \end{bmatrix} \tag{6}$$

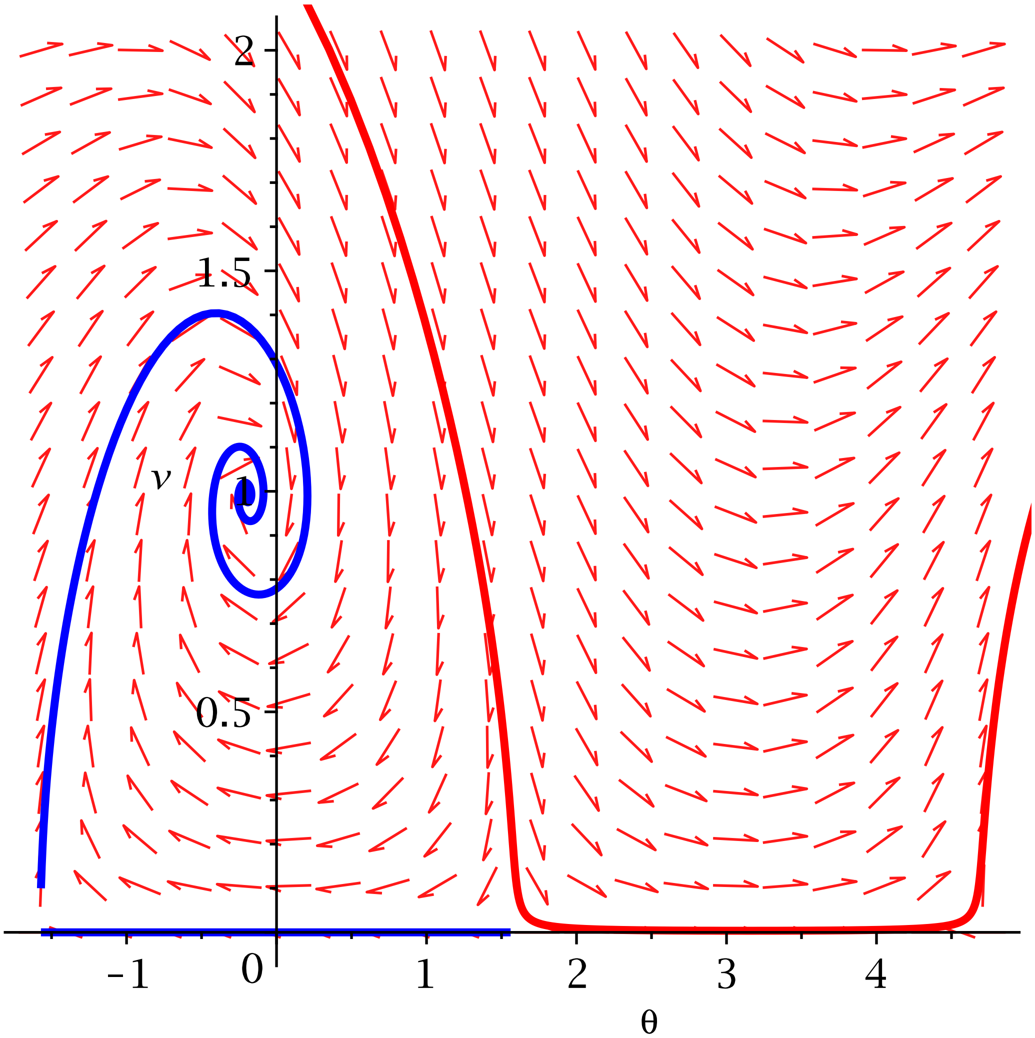
> $eval\left(Jack, \left\{\text{theta} = \frac{\text{Pi}}{2}, v = 0\right\}\right)$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1. \end{bmatrix} \tag{7}$$

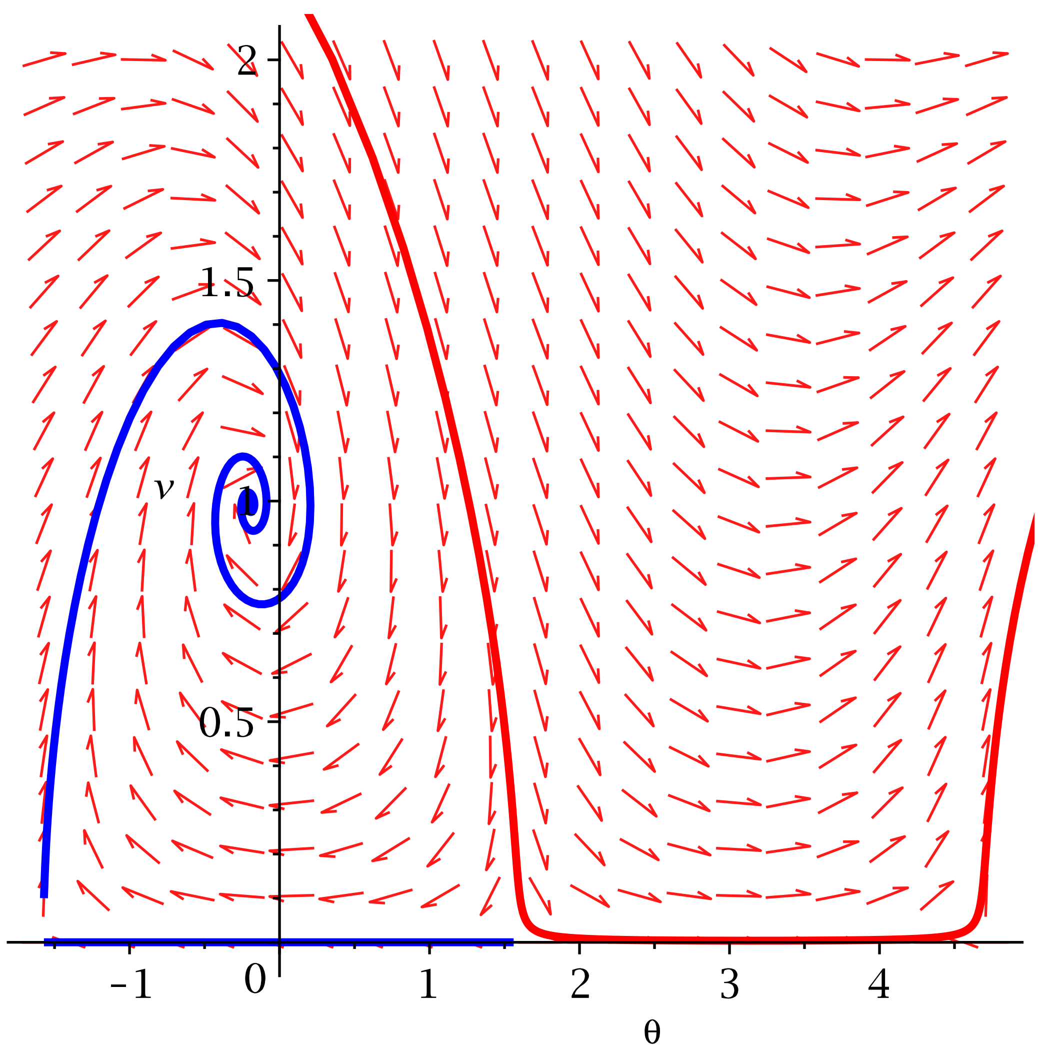
> $eval\left(Jack, \left\{\text{theta} = \frac{-\text{Pi}}{2}, v = 0\right\}\right)$

$$\begin{bmatrix} -1 & 0 \\ 0 & 1. \end{bmatrix} \tag{8}$$

> $DEplot\left(vphug, [\text{theta}, v], t = 0..50, \left[\left[v(0) = 0, \text{theta}(0) = \frac{\text{Pi}}{2} - .01\right], \left[v(0) = 0.1, \text{theta}(0) = \frac{-\text{Pi}}{2}\right], [v(0) = 2.23, \text{theta}(0) = 0]\right], \text{theta} = -\frac{\text{Pi}}{2}..\frac{3\cdot\text{Pi}}{2}, v = 0..2, linecolor = [blue, blue, red], obsrange = false, stepsize = .05\right);$



> $DEplot\left(vphug, [\theta, v], t = 0..50, \left[\left[v(0) = 0, \theta(0) = \frac{\text{Pi}}{2} - .01\right], \left[v(0) = 0.1, \theta(0) = \frac{-\text{Pi}}{2}\right], [v(0) = 2.23, \theta(0) = 0]\right], \theta = -\frac{\text{Pi}}{2} .. \frac{3 \cdot \text{Pi}}{2}, v = 0..2, linecolor = [blue, blue, red], obsrange = false, stepsize = .1\right)$



2
1.5
1
0.5
v
-1
0
1
2
3
4
θ