2029-09-05 ... the future is now!

related to problem #2

```
> f := x→x^4 - 5
```

$$f := x \mapsto x^4 - 5 \tag{1}$$

```
> factor(f(x))
```

$$x^4 - 5 \tag{2}$$

need to tell maple to adjoin the 4th root of 5 to the rationals...

```
> factor(f(x), 5^(1/4))
```

$$-\left(\sqrt{5} + x^2\right)\left(-x + 5^{1/4}\right)\left(x + 5^{1/4}\right) \tag{3}$$

the following is wrong (question wants EXACT solutions)

```
> factor(f(x), complex)
```

$$(x + 1.49534878122122)\,(x + 1.495348781\,I)\,(x - 1.495348781\,I)\,(x - 1.49534878122122) \tag{4}$$

```
> factor(f(x), {5^(1/4), I})
```

$$\left(I\,5^{1/4} - x\right)\left(I\,5^{1/4} + x\right)\left(-x + 5^{1/4}\right)\left(x + 5^{1/4}\right) \tag{5}$$

If for some reason we didn't know the roots of $x^4 = 5$, we can ask....

```
> RootOf(x^4 - 5)
```

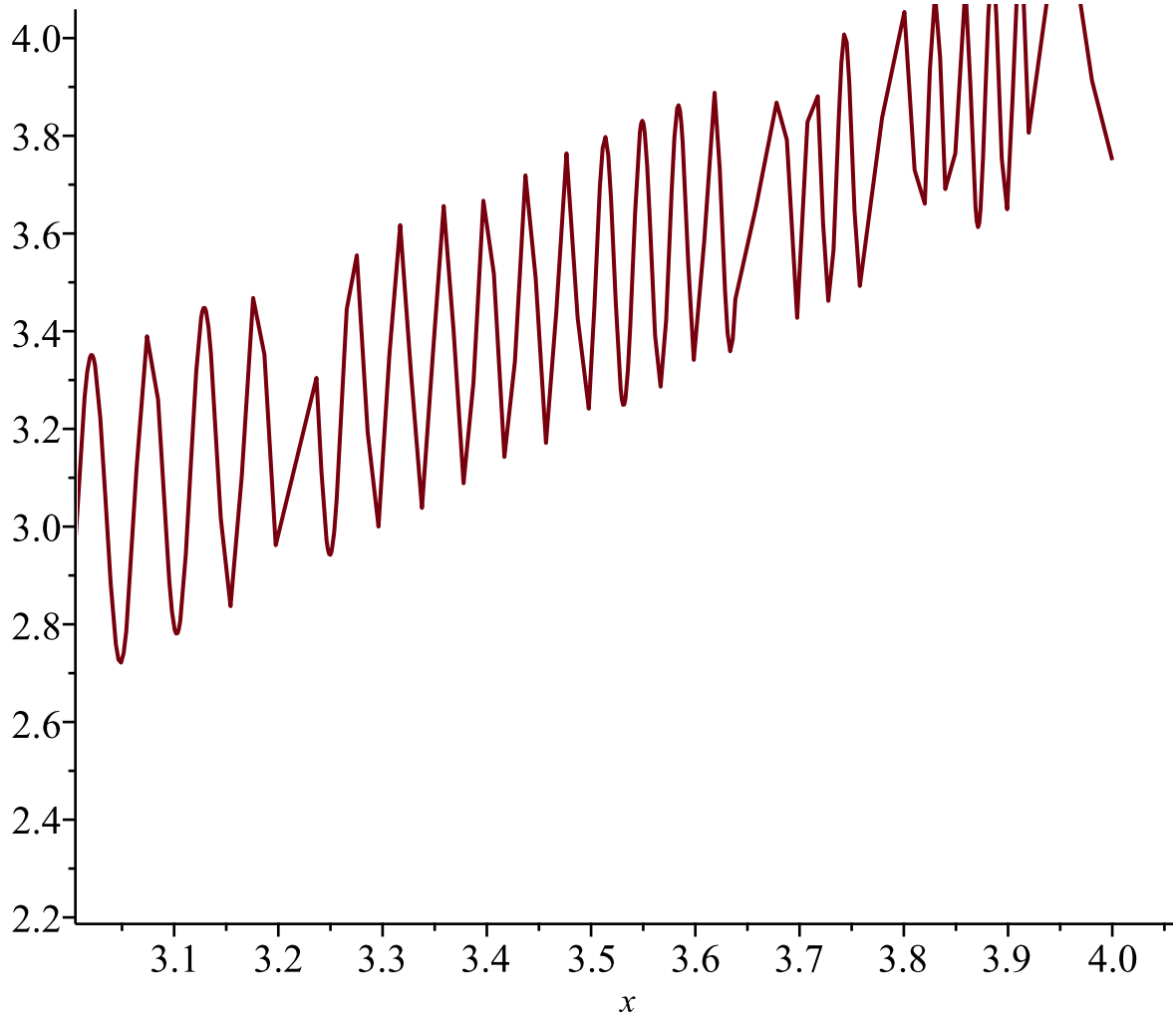
$$RootOf\left(\_Z^4 - 5\right) \tag{6}$$

```
> allvalues(%)
```

$$5^{1/4},\ I\,5^{1/4},\ -5^{1/4},\ -I\,5^{1/4} \tag{7}$$

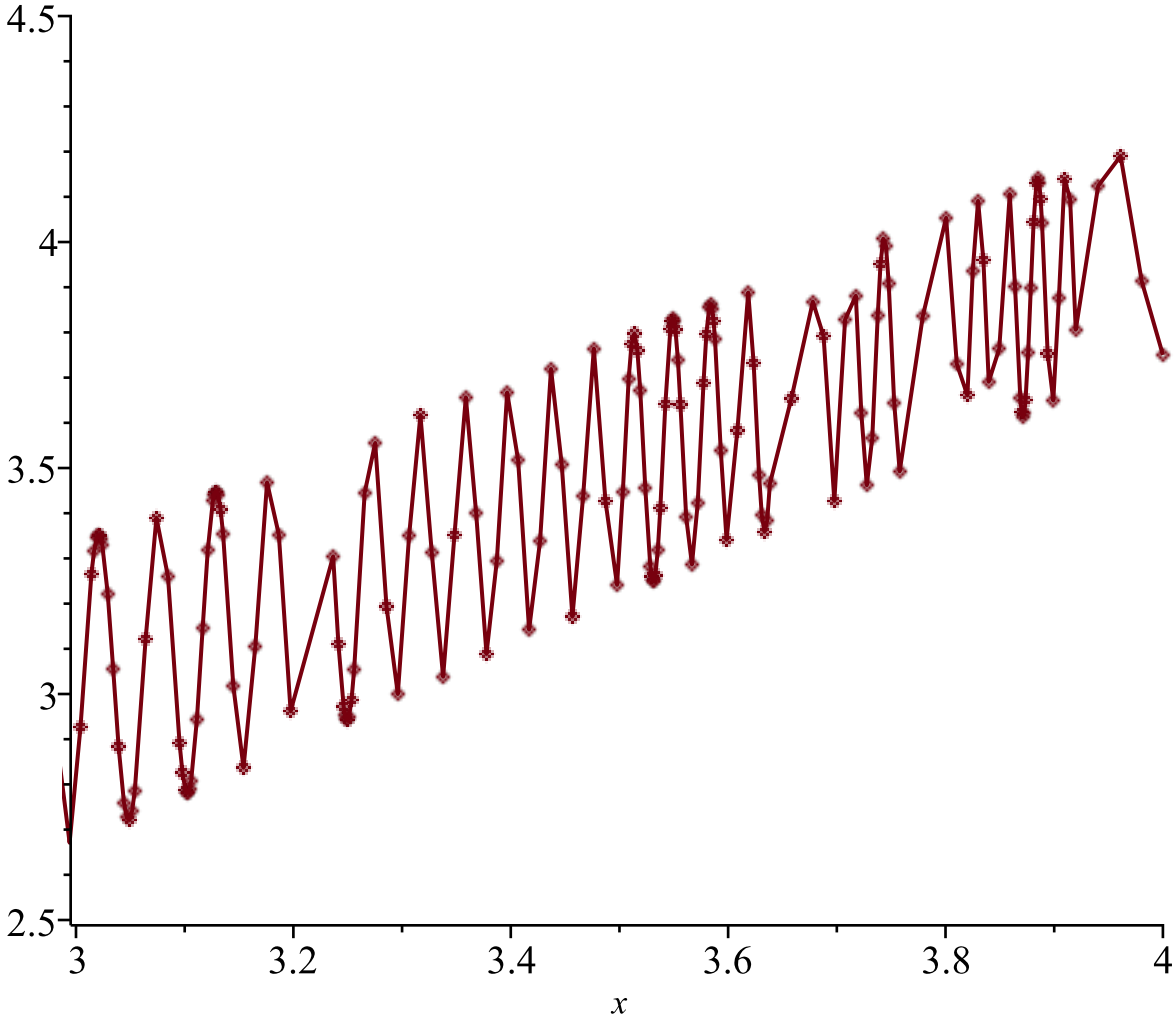
Enough of that

```
> plot(sin(x^4)/x + x, x = -4..4)
```

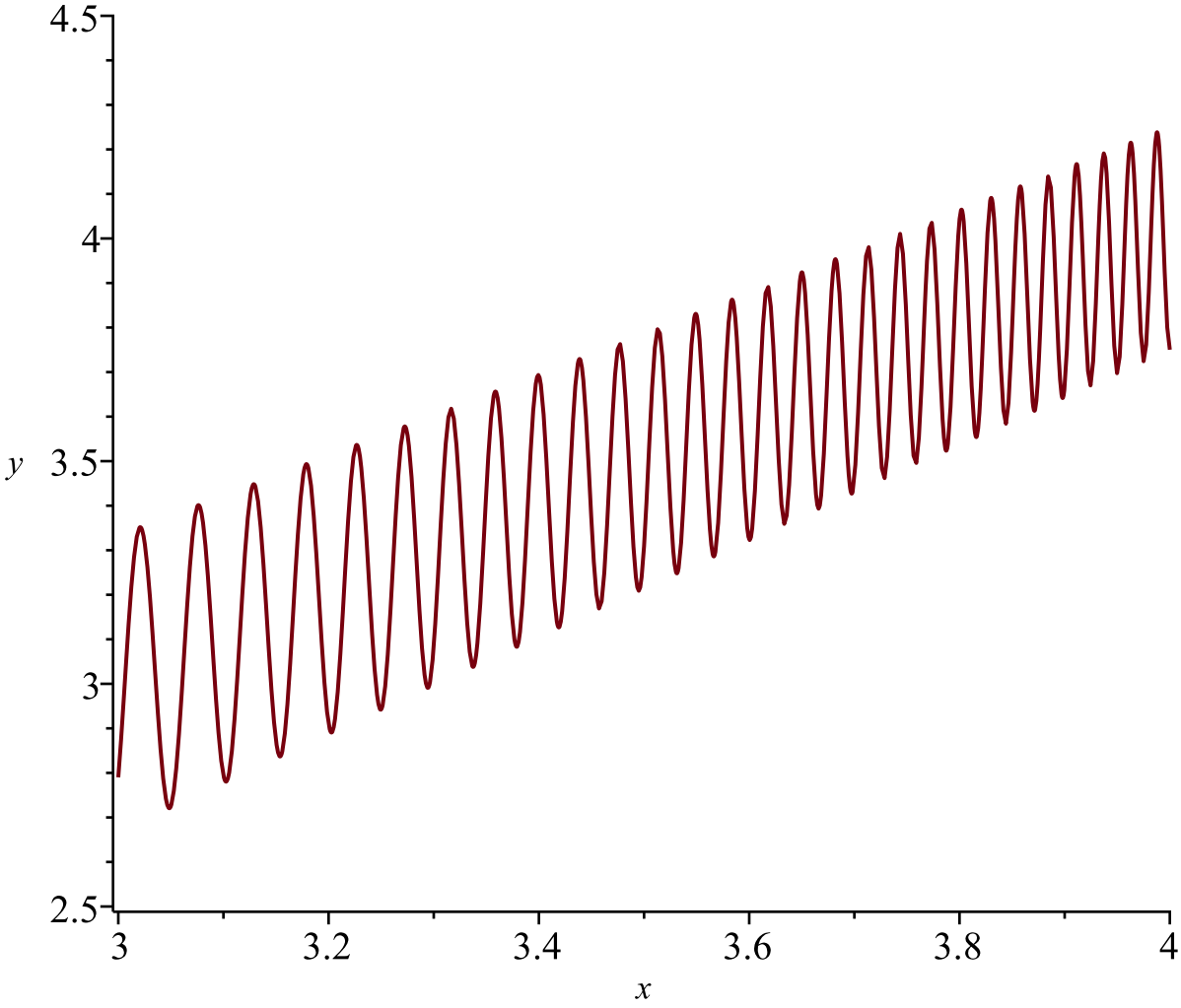


This is a sucky picture... what's wrong?

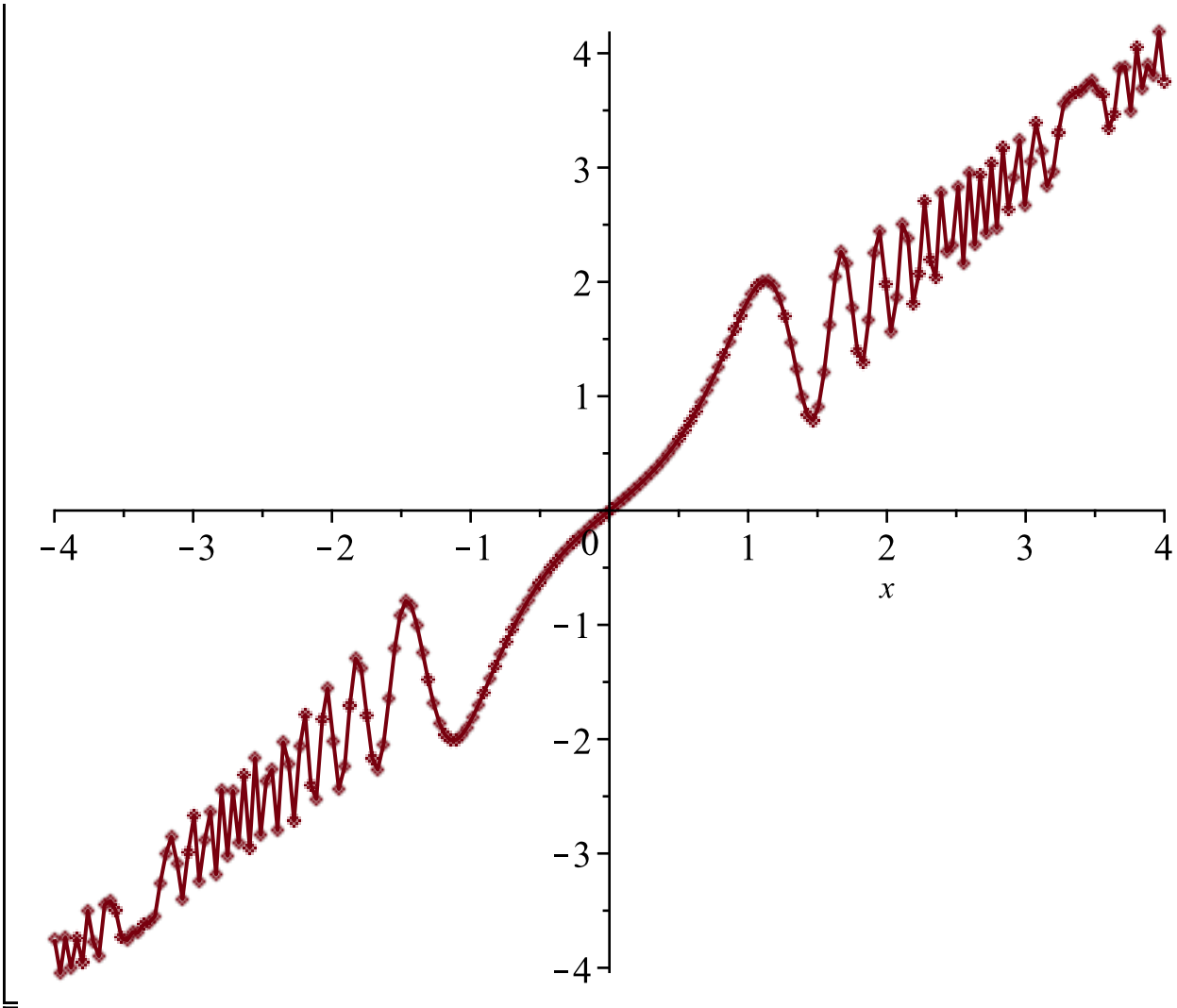
```
> plot(sin(x^4)/x + x, x = -4..4, view = [3..4, 2.5..4.5])
```



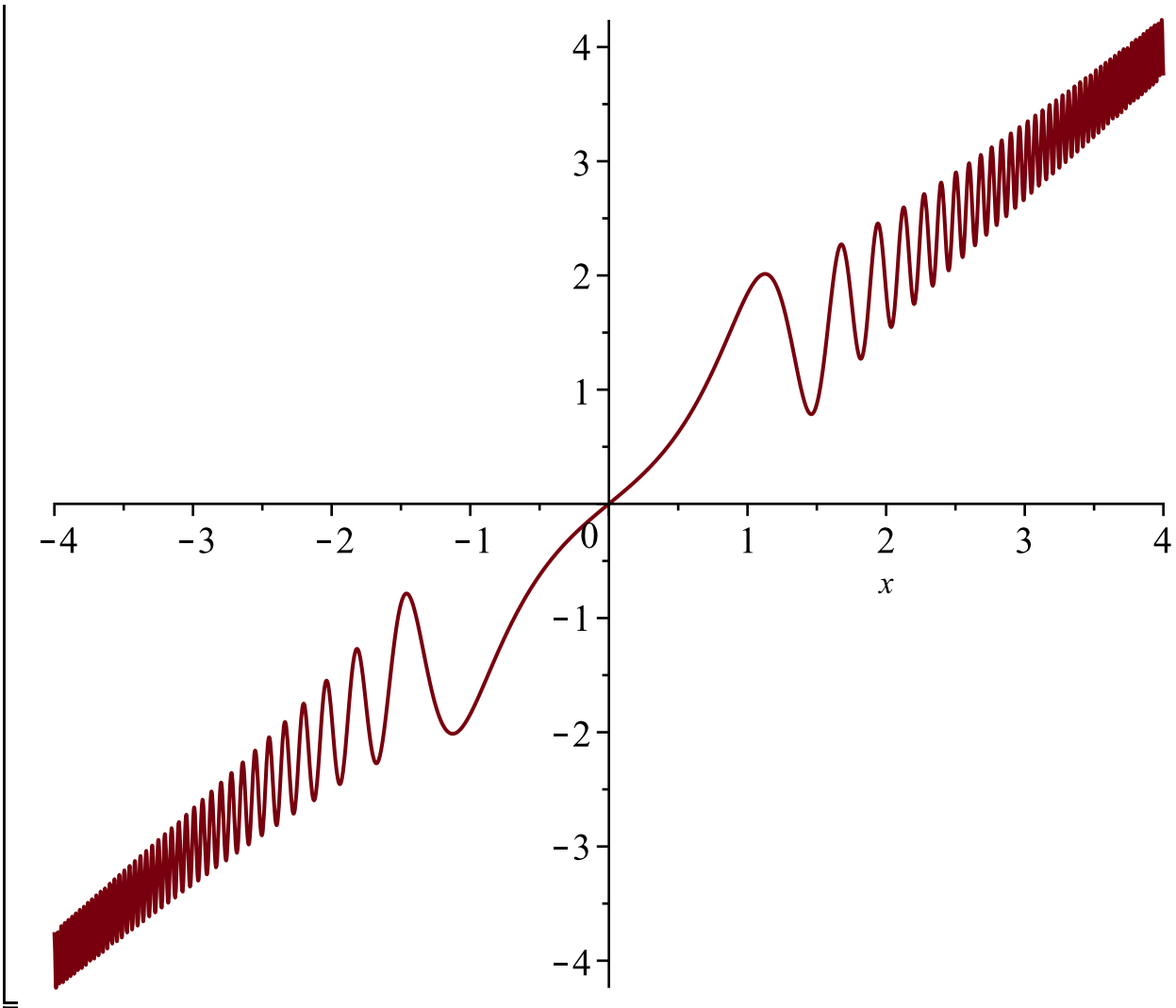
> $plot\left(\frac{\sin\left(x^4\right)}{x} + x, x = 3..4, y = 2.5..4.5\right)$



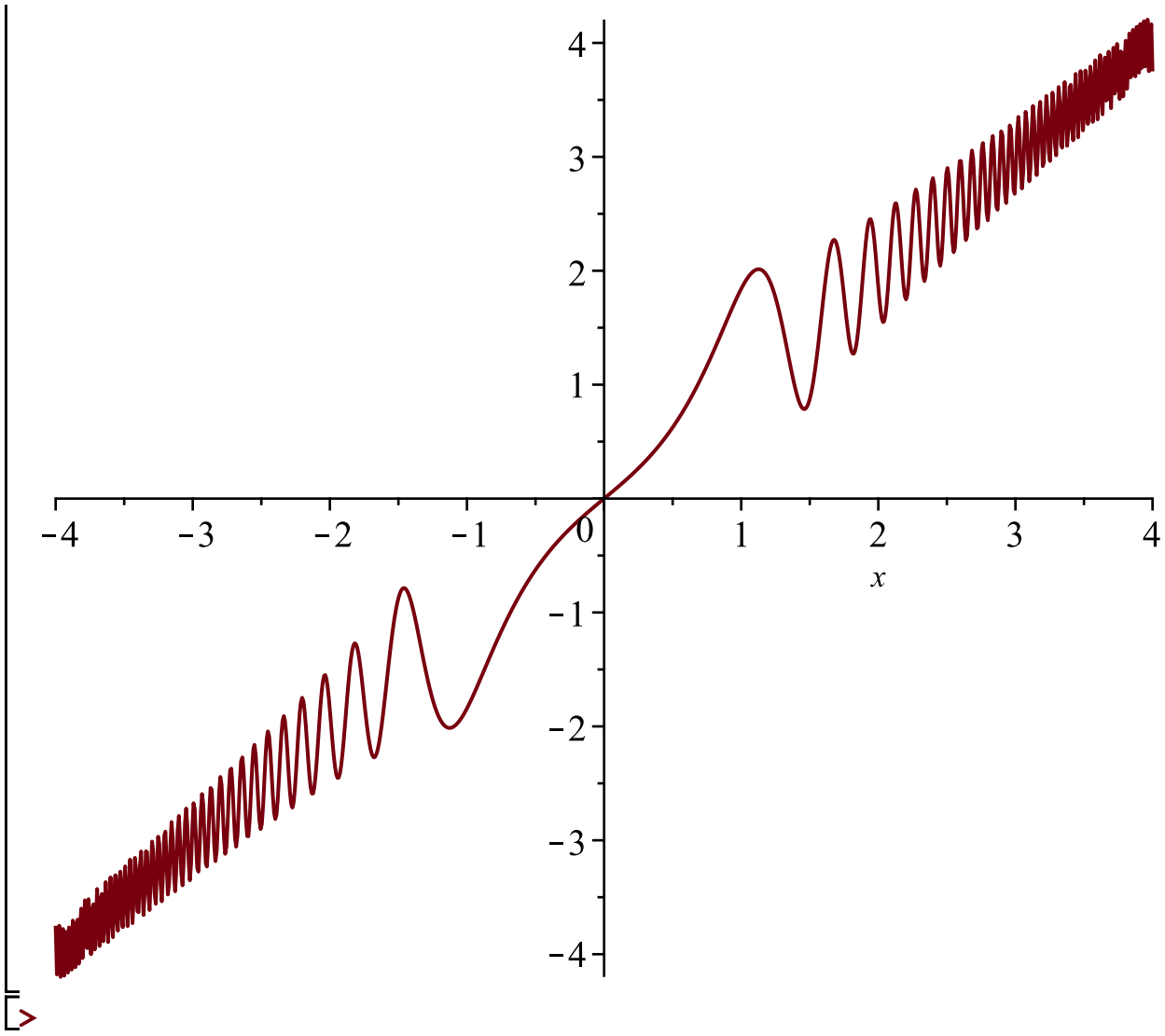
> $plot\left(\frac{\sin\left(x^4\right)}{x} + x, x = -4..4, adaptive = false, style = pointline\right)$




```
> plot(sin(x^4)/x + x, x = -4 ..4, numpoints = 900)
```




```
> plot(sin(x^4)/x + x, x = -4 ..4, numpoints = 900, adaptive = false)
```




```
>
```

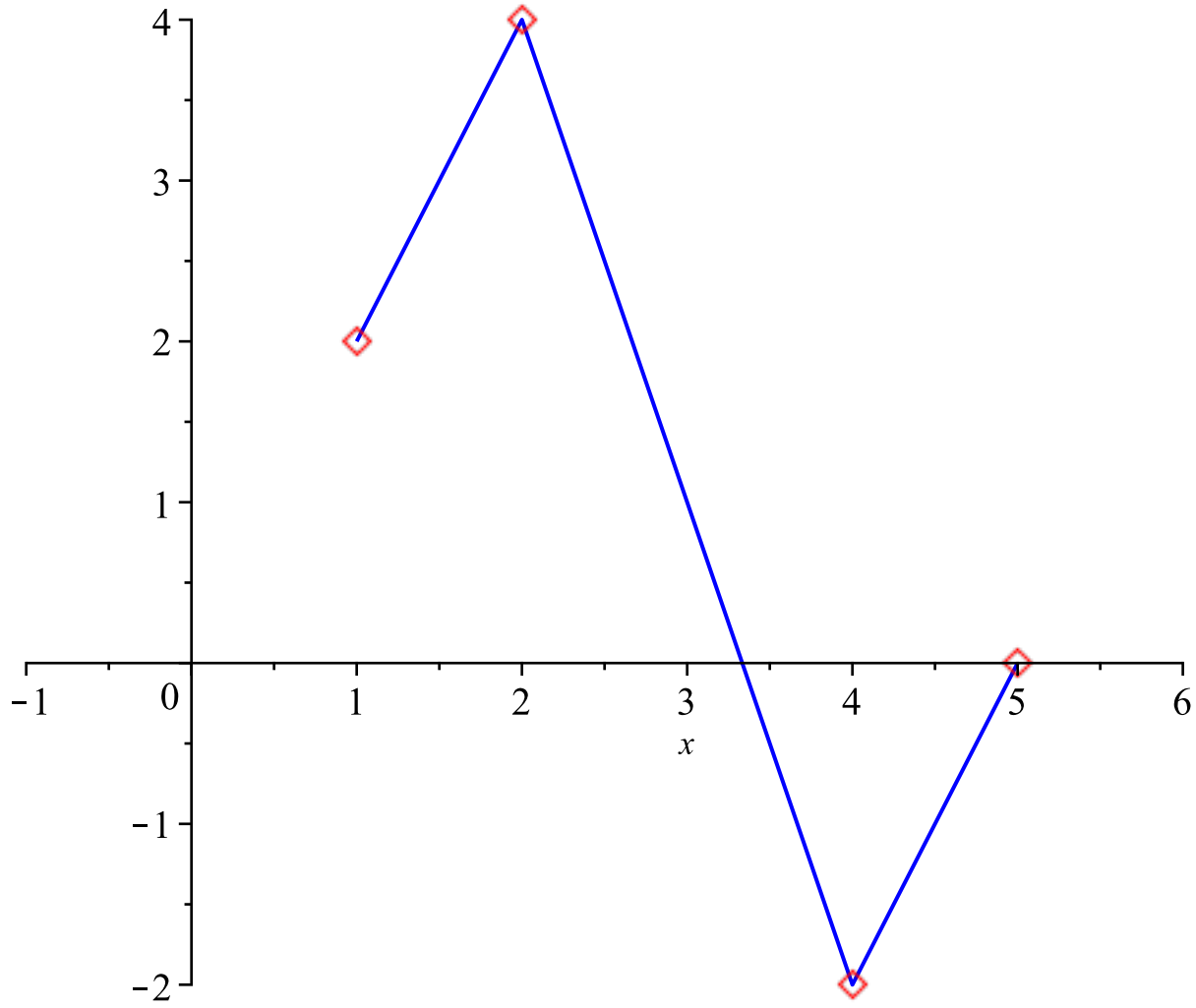
```
> data := [[1, 2], [2, 4], [4, -2], [5, 0]];
```

$$data := [[1, 2], [2, 4], [4, -2], [5, 0]] \quad \textbf{(8)}$$

```
> nops(data);
```

$$4 \quad \textbf{(9)}$$

```
> plot([data, data], x = -1 ..6,
      style = [line, point], symbolsize = 20, color = [blue, red]);
```



find the unique cubic passing through the four points.

> $cub := x \to a \cdot x^3 + b \cdot x^2 + c\,x + d;$
> $data;$

$$cub := x \mapsto a\,x^3 + b\,x^2 + c\,x + d$$
$$[[1, 2], [2, 4], [4, -2], [5, 0]] \quad \textbf{(10)}$$

want to solve 4 equations.

> $cub(1) = 2;\ cub(2) = 4;\ cub(4) = -2;\ cub(5) = 0$

$$a + b + c + d = 2$$
$$8\,a + 4\,b + 2\,c + d = 4$$
$$64\,a + 16\,b + 4\,c + d = -2$$
$$125\,a + 25\,b + 5\,c + d = 0 \quad \textbf{(11)}$$

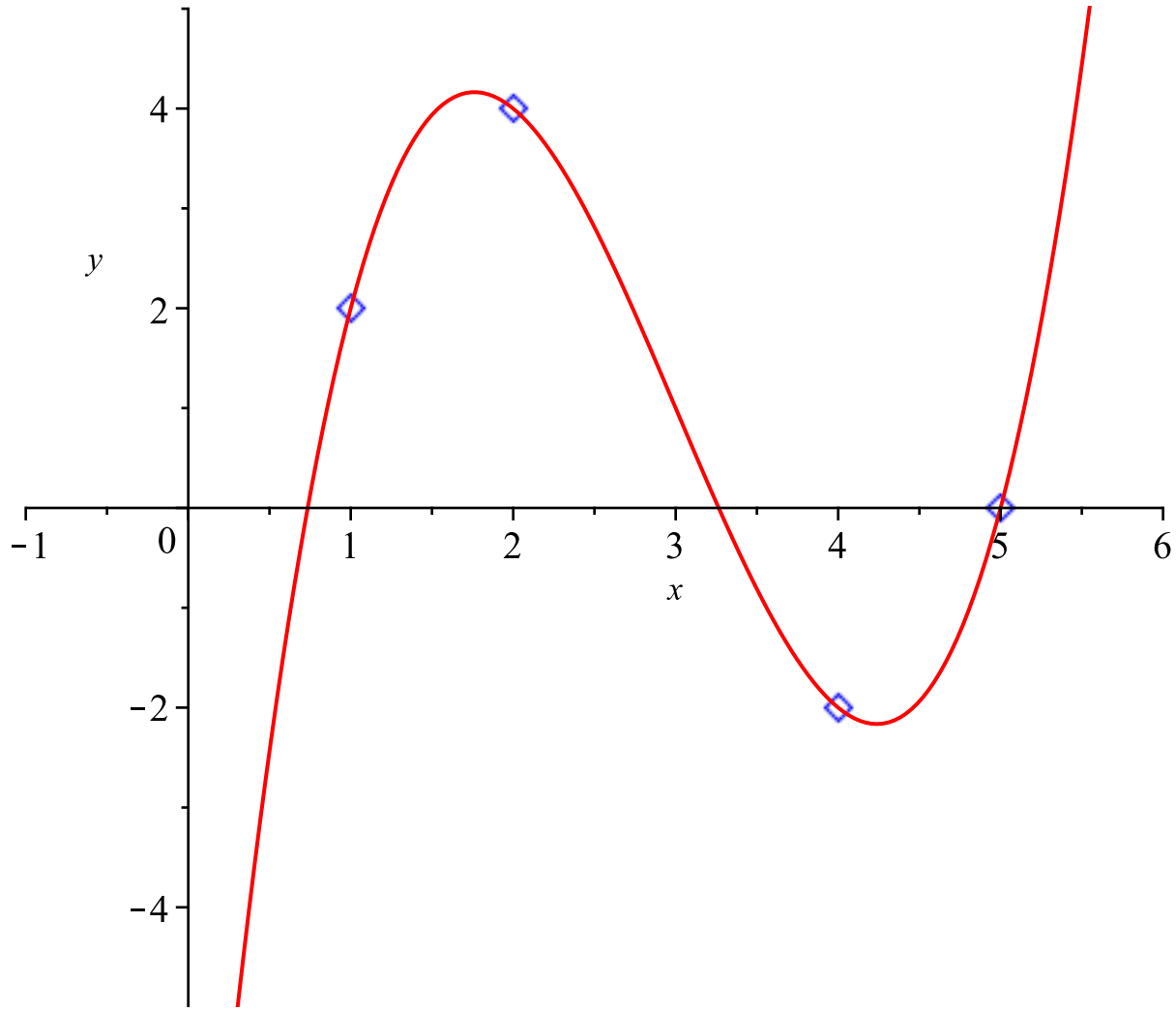
> $saul := solve(\{cub(1) = 2, cub(2) = 4, cub(4) = -2, cub(5) = 0\})$

$$saul := \left\{a = \frac{5}{6}, b = -\frac{15}{2}, c = \frac{56}{3}, d = -10\right\} \quad \textbf{(12)}$$

> $subs(saul, cub(x));$

$$\frac{5}{6}\,x^3 - \frac{15}{2}\,x^2 + \frac{56}{3}\,x - 10 \quad \textbf{(13)}$$

```
> plot([data, 5/6 x^3 - 15/2 x^2 + 56/3 x - 10], x = -1..6, y = -5..5,
      style = [point, line], symbolsize = 20, color = [blue, red]);
```



I can pick off the x and y values of data...

```
> data[3], " is", data[3][1], data[3][2]
```

$$[4, -2], \text{ is}, 4, -2 \quad \textbf{(14)}$$

```
> cub(data[3][1]) = data[3][2]
```

$$64\,a + 16\,b + 4\,c + d = -2 \quad \textbf{(15)}$$

```
> {seq(
      cub(data[i][1]) = data[i][2],
      i = 1..nops(data)
    )}
```

$$\{a + b + c + d = 2,\ 8\,a + 4\,b + 2\,c + d = 4,\ 64\,a + 16\,b + 4\,c + d = -2,\ 125\,a + 25\,b + 5\,c + d = 0\} \quad \textbf{(16)}$$

```
> wrong := x -> a x^3
```

$$wrong := x \mapsto a\,x^3 \quad \textbf{(17)}$$

```
> wrong(3)
```

$$ax^3 \tag{18}$$

```
> a*x^3
```

$$a\,x^3 \tag{19}$$

```
> eqlist := dat→{seq(
      cub(dat[i][1]) = dat[i][2],
      i = 1..nops(dat)
    )}
Warning, `i` is implicitly declared local to procedure `eqlist`
```

$$eqlist := dat \mapsto \left\{seq\left(cub\left(\left(dat_i\right)_1\right) = \left(dat_i\right)_2,\ i = 1..nops(dat)\right)\right\} \tag{20}$$

```
> eqlist(data)
```

$$\{a+b+c+d=2,\ 8a+4b+2c+d=4,\ 64a+16b+4c+d=-2,\ 125a+25b+5c+d=0\} \tag{21}$$

```
> eqlist( [[-2, 3], [0, 4], [5, 1], [7, 8]])
```

$$\{d=4,\ -8a+4b-2c+d=3,\ 125a+25b+5c+d=1,\ 343a+49b+7c+d=8\} \tag{22}$$

```
> solve(eqlist(data))
```

$$\left\{a=\frac{5}{6},\ b=-\frac{15}{2},\ c=\frac{56}{3},\ d=-10\right\} \tag{23}$$

```
> subs(%, cub(x))
```

$$\frac{5}{6}x^3-\frac{15}{2}x^2+\frac{56}{3}x-10 \tag{24}$$

```
> makecub := data→subs( solve(eqlist(data)), cub(x))
```

$$makecub := data \mapsto subs(solve(eqlist(data)), cub(x)) \tag{25}$$

```
> makecub(data)
```

$$\frac{5}{6}x^3-\frac{15}{2}x^2+\frac{56}{3}x-10 \tag{26}$$

```
> unapply transforms an expression into a function.
> f := unapply(makecub(data), x)
```

$$f := x \mapsto \frac{5}{6}x^3-\frac{15}{2}x^2+\frac{56}{3}x-10 \tag{27}$$

```
> stuff := [first, second, third]
```

$$stuff := [first, second, third] \tag{28}$$

```
> stuff[2]
```

$$second \tag{29}$$

```
> nonsense := [frs, [sec1, sec2, sec3], [apple, pear]]
```

$$nonsense := [frs, [sec1, sec2, sec3], [apple, pear]] \tag{30}$$

```
> nonsense[2]
```

$$[sec1, sec2, sec3] \tag{31}$$

```
> nonsense[2, 3]
```

$$sec3 \tag{32}$$

```
> nonsense[3, 3]
```

Error, invalid subscript selector

```
> nonsense[1, 3]
```

Error, invalid subscript selector

```
> nonsense[1][3]
```

$$frs_3 \tag{33}$$

maybe we want a looooop (to make a newline without submitting, do shift-enter

```
> for i from 1 to 3 do
    nonsense[i];
  od
```

$$frs$$
$$[sec1, sec2, sec3]$$
$$[apple, pear] \tag{34}$$

Actually, this is polynomial interpolation, already built in.

```
> ?PolynomialInterpolation
> with(CurveFitting)
```

$$[ArrayInterpolation, BSpline, BSplineCurve, Interactive, LeastSquares, Lowess, PolynomialInterpolation, RationalInterpolation, Spline, ThieleInterpolation] \tag{35}$$

```
> PolynomialInterpolation(data, x)
```

$$\frac{5}{6} x^3 - \frac{15}{2} x^2 + \frac{56}{3} x - 10 \tag{36}$$

```
>
```