

1. Plot the function $f(x) = 2 \sin x - x^3 - \frac{1}{5}$, for $x \in [-4, 4]$.
Find all the zeros of the function with an accuracy of 20 decimal digits. Hint: See Digits, fsolve

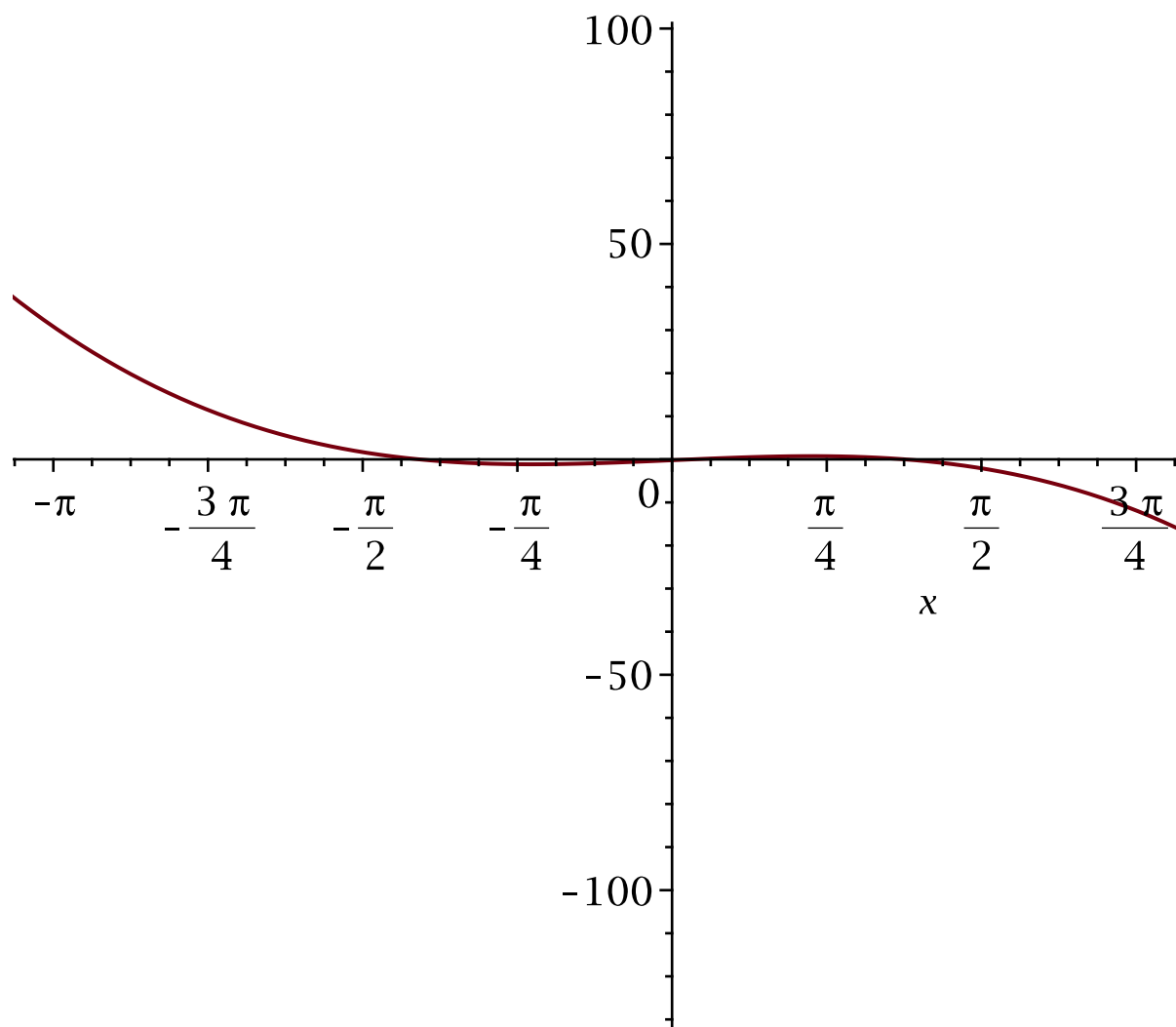
> $f := 2 \sin(x) - x^3 - \frac{1}{5};$

$$f := 2 \sin(x) - x^3 - \frac{1}{5} \quad (1)$$

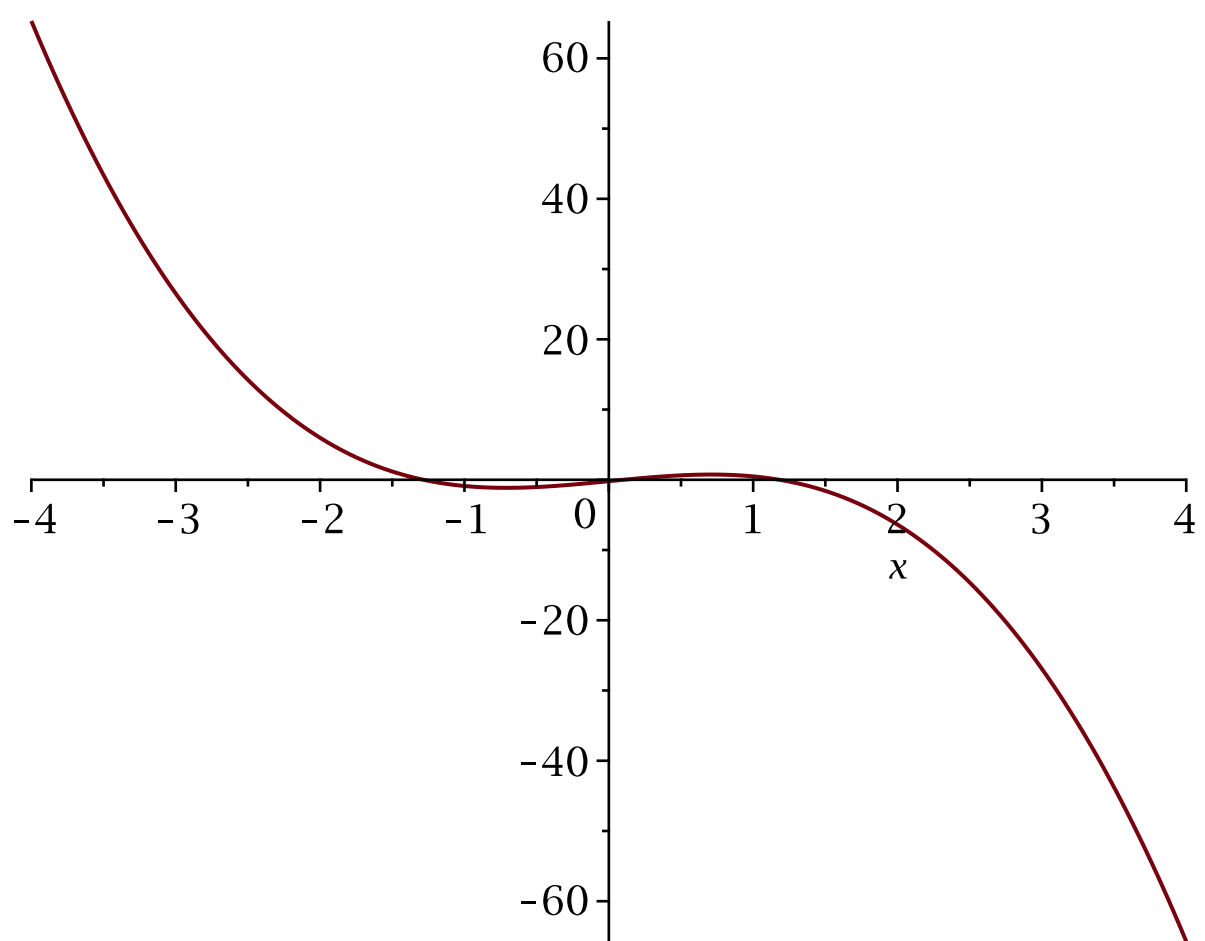
> f

$$2 \sin(x) - x^3 - \frac{1}{5} \quad (2)$$

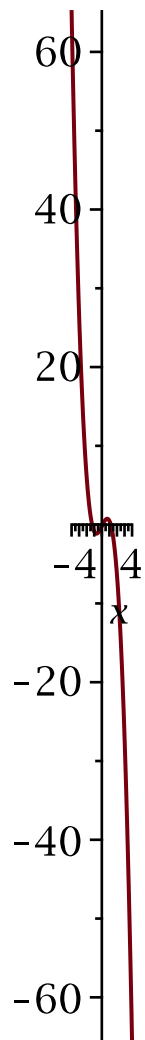
> $plot(f);$



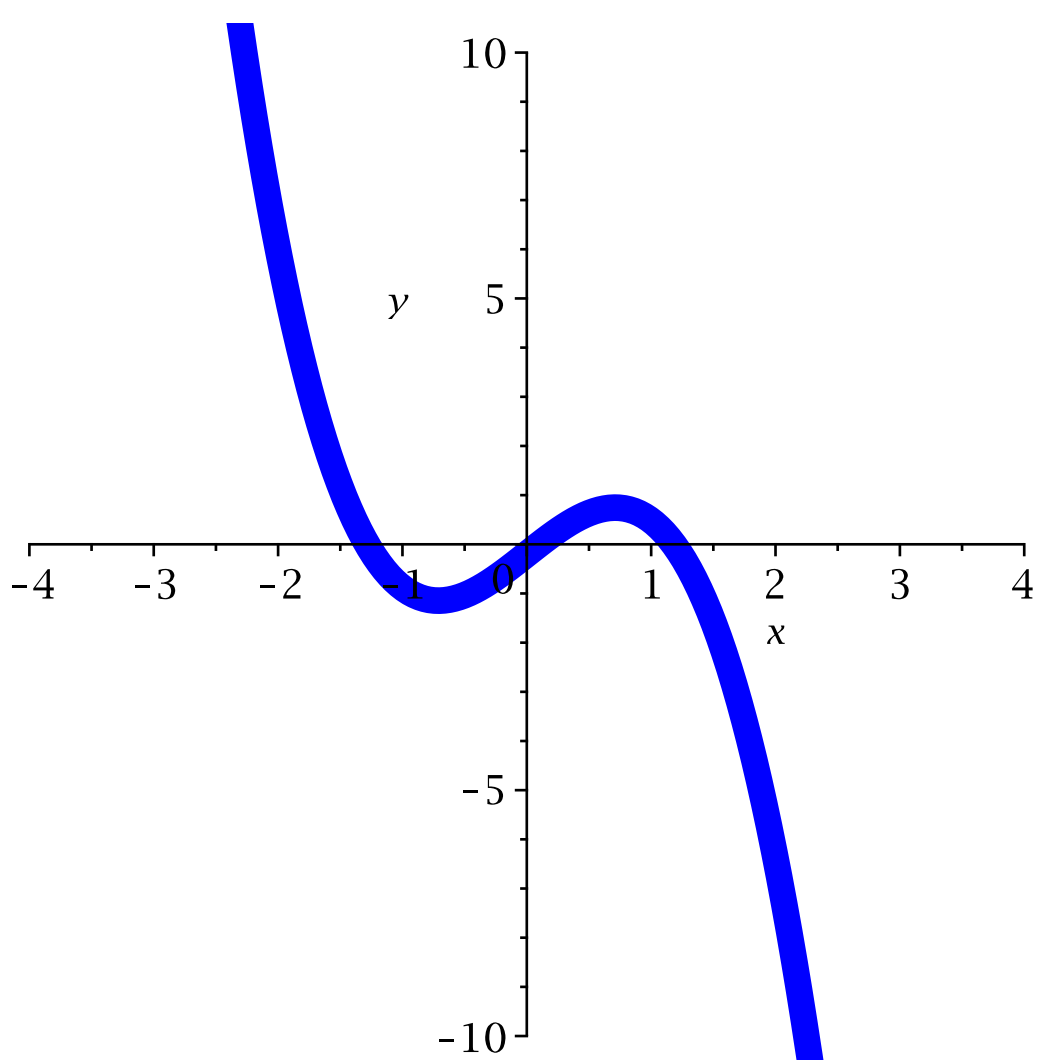
> $plot(f, x = -4..4);$



```
> plot(f, x = -4..4, scaling = constrained);
```



```
> plot(f, x = -4..4, y = -10..10, color = blue, thickness = 10)
```



> Here I cut and pasted stuff to answer part of the first exercise.

> f

$$2 \sin(x) - x^3 - \frac{1}{5} \quad (3)$$

> $f(1.2)$

$$2 \sin(x)(1.2) - x(1.2)^3 - \frac{1}{5} \quad (4)$$

> $g := x^2$

$$g := x^2 \quad (5)$$

> $g(-1)$

$$x(-1)^2 \quad (6)$$

[also wrong, but maple is being nice.

> $g(x) := x^2$

$$g := x \mapsto x^2 \quad (7)$$

here I typed $g:=x \rightarrow x^2$

> $g := x \rightarrow x^2$

$$g := x \mapsto x^2 \quad (8)$$

> $g(3);$

$$9 \quad (9)$$

> $g(2.6);$

$$6.76 \quad (10)$$

I might try this, but it fails.

> $newf := x \rightarrow f;$

$$newf := x \mapsto f \quad (11)$$

> $newf(3)$

$$2 \sin(x) - x^3 - \frac{1}{5} \quad (12)$$

> $anotherTry := x \rightarrow 2 \sin(x) - x^3 - \frac{1}{5}$

$$anotherTry := x \mapsto 2 \sin(x) - x^3 - \frac{1}{5} \quad (13)$$

> $anotherTry(2.1);$

$$-7.734581267 \quad (14)$$

I don't want to copy/paste or retype. So.....
 spoze I just want to plug 1.2 into the expression f

> $eval(f, x = 1.3)$

$$-0.4698836290 \quad (15)$$

> $subs(x = 1.3, f)$

$$2 \sin(1.3) - 2.397000000 \quad (16)$$

> $subs(x = y^2, f);$

$$2 \sin(y^2) - y^6 - \frac{1}{5} \quad (17)$$

> $eval(f, x = 1);$

$$2 \sin(1) - \frac{6}{5} \quad (18)$$

> $evalf(\%)$

$$0.482941970 \quad (19)$$

> f

$$2 \sin(x) - x^3 - \frac{1}{5} \quad (20)$$

```
> fAsAFunc := unapply(f, x)
      fAsAFunc := x ↦ 2 sin(x) - x3 -  $\frac{1}{5}$  (21)
```

```
> fAsAFunc(x) is way too long, let's just turn f into a function.
```

```
> f := unapply(f, x)
      f := x ↦ 2 sin(x) - x3 -  $\frac{1}{5}$  (22)
```

```
> f(1.02);
      0.4430080440 (23)
```

```
> solve(f(x) = 0, x)
      RootOf(5 _Z3 - 10 sin(_Z) + 1) (24)
```

```
> solve(y = x3 - 1, x)
      (y + 1)1/3, - $\frac{(y+1)^{1/3}}{2} + \frac{I\sqrt{3}(y+1)^{1/3}}{2}$ , - $\frac{(y+1)^{1/3}}{2} - \frac{I\sqrt{3}(y+1)^{1/3}}{2}$  (25)
```

```
> solve(3 = x3 - 1, x)
      22/3, - $\frac{2^{2/3}}{2} + \frac{I\sqrt{3} 2^{2/3}}{2}$ , - $\frac{2^{2/3}}{2} - \frac{I\sqrt{3} 2^{2/3}}{2}$  (26)
```

```
> evalf(%);
      1.587401052, -0.7937005260 + 1.374729638 I, -0.7937005260
      - 1.374729638 I (27)
```

```
> argument(%[2]);
      2.094395102 (28)
```

```
> sols := (solve(3 = x3 - 1, x))
      sols := 22/3, - $\frac{2^{2/3}}{2} + \frac{I\sqrt{3} 2^{2/3}}{2}$ , - $\frac{2^{2/3}}{2} - \frac{I\sqrt{3} 2^{2/3}}{2}$  (29)
```

```
> argument(sols[2]);
       $\frac{2\pi}{3}$  (30)
```

```
> abs(sols[2]);
      22/3 (31)
```

```
> evalf(%);
      1.587401052 (32)
```

```
> argument(sols);
      Error. (in argument) expecting 1 argument, got 3
```

```
> map(argument, [sols])
      [0,  $\frac{2\pi}{3}$ , - $\frac{2\pi}{3}$ ] (33)
```

```
> map(abs, [sols])
      (34)
```

$$[2^{2/3}, 2^{2/3}, 2^{2/3}] \quad (34)$$

$$\begin{aligned} &> \text{map}(x \rightarrow [\text{abs}(x), \text{argument}(x)], \{\text{sols}\}); \\ &\quad \left\{ [2^{2/3}, 0], \left[2^{2/3}, -\frac{2\pi}{3} \right], \left[2^{2/3}, \frac{2\pi}{3} \right] \right\} \end{aligned} \quad (35)$$

$$\begin{aligned} &> \text{fsolve}(f(x) = 0, x) \\ &\quad 0.1006802788 \end{aligned} \quad (36)$$

$$\begin{aligned} &> \text{Digits} := 20; \\ &\quad \text{Digits} := 20 \end{aligned} \quad (37)$$

$$\begin{aligned} &> \text{fsolve}(f(x) = 0, x) \\ &\quad 0.10068027882300758308 \end{aligned} \quad (38)$$

$$\begin{aligned} &> \text{fsolve}(f(x) = 0, x) : \\ &> \text{argument}(1 + I); \\ &\quad \frac{\pi}{4} \end{aligned} \quad (39)$$

$$\begin{aligned} &> \text{argument}(-1 + I) \\ &\quad \frac{3\pi}{4} \end{aligned} \quad (40)$$

$$\begin{aligned} &> \text{sols}, \\ &\quad 2^{2/3}, -\frac{2^{2/3}}{2} + \frac{I\sqrt{3} 2^{2/3}}{2}, -\frac{2^{2/3}}{2} - \frac{I\sqrt{3} 2^{2/3}}{2} \end{aligned} \quad (41)$$

$$\begin{aligned} &> [\text{sols}] \\ &\quad \left[2^{2/3}, -\frac{2^{2/3}}{2} + \frac{I\sqrt{3} 2^{2/3}}{2}, -\frac{2^{2/3}}{2} - \frac{I\sqrt{3} 2^{2/3}}{2} \right] \end{aligned} \quad (42)$$

$$\begin{aligned} &> \text{map}(x \rightarrow x^3, \%); \\ &\quad \left[4, \left(-\frac{2^{2/3}}{2} + \frac{I\sqrt{3} 2^{2/3}}{2} \right)^3, \left(-\frac{2^{2/3}}{2} - \frac{I\sqrt{3} 2^{2/3}}{2} \right)^3 \right] \end{aligned} \quad (43)$$

$$\begin{aligned} &> g := x \rightarrow x^3 \\ &\quad g := x \mapsto x^3 \end{aligned} \quad (44)$$

$$\begin{aligned} &> \text{map}(g, [\text{sols}]) \\ &\quad \left[4, \left(-\frac{2^{2/3}}{2} + \frac{I\sqrt{3} 2^{2/3}}{2} \right)^3, \left(-\frac{2^{2/3}}{2} - \frac{I\sqrt{3} 2^{2/3}}{2} \right)^3 \right] \end{aligned} \quad (45)$$

$$\begin{aligned} &> \text{map}(f, [\text{sols}]); \\ &\quad \left[2 \sin(2^{2/3}) - \frac{21}{5}, -2 \sin\left(\frac{2^{2/3}}{2} - \frac{I\sqrt{3} 2^{2/3}}{2}\right) - \left(-\frac{2^{2/3}}{2} + \frac{I\sqrt{3} 2^{2/3}}{2}\right)^3 - \frac{1}{5}, \right. \\ &\quad \left. -2 \sin\left(\frac{2^{2/3}}{2} + \frac{I\sqrt{3} 2^{2/3}}{2}\right) - \left(-\frac{2^{2/3}}{2} - \frac{I\sqrt{3} 2^{2/3}}{2}\right)^3 - \frac{1}{5} \right] \end{aligned} \quad (46)$$

$$\begin{aligned} &> \text{fsolve}(f(x) = 0, x) \\ &\quad 0.10068027882300758308 \end{aligned} \quad (47)$$

```

> f(%)
0. (48)
=
> Digits := 10
Digits := 10 (49)
=
> fsolve(f(x) = 0, x);
0.1006802788 (50)
=
> f(%);
-1. 10-10 (51)
=
> Digits := 30
Digits := 30 (52)
=
> f(0.10068027882300758308)
-8.221234418 10-21 (53)
=
> ?Digits
> f(y)
2 sin(y) - y3 -  $\frac{1}{5}$  (54)
=
> fex := 2 sin(x) - x3 -  $\frac{1}{5}$ 
fex := 2 sin(x) - x3 -  $\frac{1}{5}$  (55)
=
> s := fsolve(fex = 0, x)
s := 0.100680278823007583084195657982 (56)
=
> f := x → 2 sin(x) - x3 -  $\frac{1}{5}$ 
f := x ↦ 2 sin(x) - x3 -  $\frac{1}{5}$  (57)
=
> f(s)
-1. 10-30 (58)
=
> fex(s)
2 sin(x)(0.100680278823007583084195657982)
- x(0.100680278823007583084195657982)3 -  $\frac{1}{5}$  (59)
=
> eval(fex, x = s)
-1. 10-30 (60)
>

```