

## Math 331, Fall 2002: Problems 11-12

**NOTE:** *Each exercise is worth 10 points and can be turned in at any time before its “expiration date”. At the end of the semester, I will expect you to have turned in at least 2/5 of the exercises assigned. If you do more, I will pick your best grades. If you do less, the missing grades will be counted as zeros. Altogether, these will count the same as one project.*

11. (*expires 10/14*) Following Section 4 of the notes, prove that if we describe the circle of center  $(a, b)$  and radius  $r$  using the parameters  $(a, b, k)$ , with  $k = a^2 + b^2 - r^2$ , rather than the more natural parameters  $(a, b, r)$ , then the error function  $H(a, b, k) = E(a, b, \sqrt{a^2 + b^2 - k})$  is quadratic in  $a, b$  and  $k$ . What does this imply about the number of critical points?
12. (*expires 10/14*) With reference to Problem #11, show that, for  $r > 0$ , the transformation  $(a, b, r) \mapsto (a, b, k)$  is a valid change of variables, that is, it is one-to-one. This should help you prove that  $E(a, b, r)$  has only one “physical” critical point, which is a minimum, and is mapped, through the transformation, into the unique critical point of  $H(a, b, k)$ .