Problem	1	2	3	4	5	**Bonus:**	**Total:**
Points	**15**	**13**	**12**	**5**	**5**	**10**	**50+10**
Scores							

MAT 310 – LINEAR ALGEBRA – FALL 2004

Name: ______________________ **Id. #:** **Lecture #:**

Test 1 (September 24 / 50 minutes)

There are 5 problems worth 50 points total and a bonus problem worth up to 10 points. Show all work. Always indicate carefully what you are doing in each step; otherwise it may not be possible to give you appropriate partial credit.

1. [15 points] Consider the homogeneous system of linear equations

$$\begin{aligned} x_1 + \ \ x_2 + 2x_3 - 2x_4 &= 0 \\ x_1 - 5x_2 - \ \ x_3 + 7x_4 &= 0 \\ x_1 - \ \ x_2 + \ x_3 + \ x_4 &= 0 \end{aligned}$$

(a) [3 points] Write down the matrices A, X, and O for which the system is in matrix form $AX = O$.

(b) [6 points] Using the Gauss-Jordan algorithm, compute the row-reduced echelon matrix R which is row equivalent to A.

(c) [6 points] Use (b) to find all solutions of the above system.

2. [14 points] Let

$$A = \begin{bmatrix} 2 & -2 & 1 \\ 3 & -1 & 2 \\ 1 & -3 & 0 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}.$$

(a) [10 points] Apply row reduction to the augmented matrix of A and determine exactly for which triples (y_1, y_2, y_3) the system $AX = Y$ has a solution.

(b) [3 points] Write down an explicit right hand side column Y for which the system $AX = Y$ has no solution.

3. [12 points] Consider

$$A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}.$$

(a)[8 points] By any method, argue that A is invertible and compute A^{-1}.

(b)[4 points] Write A as a product of elementary matrices.

4. [5 points] Find an example of 2×2 matrices A, B for which it is *not* true that $(A+B)^2 = A^2 + 2AB + B^2$. [Can you give a condition for A and B, so the last matrix formula would hold? You'll get 5 extra points for the right answer.]

5. [5 points] Let A be an $n \times n$ matrix. Show that A is invertible if and only if A^2 is invertible.

Bonus Problem [up to 10 points] Consider the equation $X^2 = -I$ for 2×2 matrices. There are solutions with real coefficient. Discover as many as you can. Can you find infinitely many? Give some more additional solutions with complex coefficients. [You might also want to observe that if P is invertible 2×2, then if X solves, so does PXP^{-1}. Why?]