MATH 308 First Midterm

March 9, 2011

Name: ______________________ **ID:** __________________

Question:	1	2	3	4	5	Total
Points:	16	8	23	15	15	77
Score:						

There are 5 problems on 6 pages in this exam (not counting the cover sheet). Make sure that you have them all.

You **may use a calculator** if you wish, provided your calculator does not do calculus. However, it is unlikely to be of much help.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, extra papers, and discussions with friends are not permitted.**

If you have one, use of a time machine to travel to the future to check your answers is permitted, although you must subsequently allow me to use the machine to retroactively change the test. You will be responsible for any temporal paradoxes which may result.

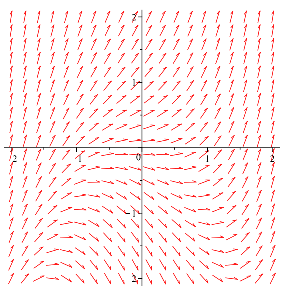
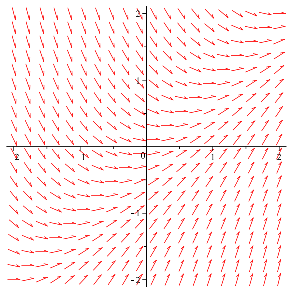
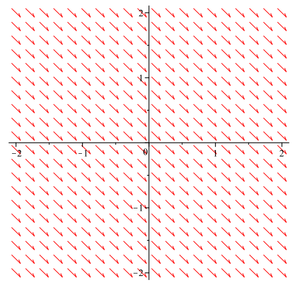
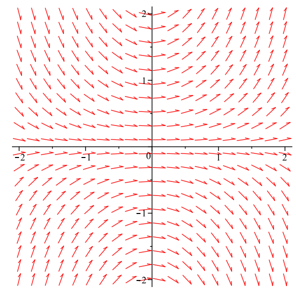
You have about 79 minutes and 47 seconds to complete this exam.

When you complete this exam, if there is sufficient time it is strongly recommended that you go back and reexamine your work, both on this exam and in your life up until now, for any errors that you may have made.

1. Consider the differential equation

$$\frac{dy}{dt} = t - y$$

$\boxed{\text{3 pts.}}$ (a) Circle the direction field below which corresponds to this equation.

$\boxed{\text{3 pts.}}$ (b) Based on your answer above, if $y(-1) = 0$, circle the statement below which is true.

A. $y(2) < -1$ B. $y(2) \approx 0$ C. $y(2) \approx 1$ D. $y(2) > 2$

$\boxed{\text{10 pts.}}$ (c) Find a function $y(t)$ such that $\dfrac{dy}{dt} = t - y$ and $y(-1) = 0$.

$\boxed{\text{8 pts.}}$ 2. The matrix $A = \begin{pmatrix} 11 & 3 & -3 \\ 8 & 6 & -8 \\ 5 & -5 & 3 \end{pmatrix}$ has eigenvectors $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$, and $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

Find a matrix P such that $C = PAP^{-1}$ is a diagonal matrix[1]; your answer should list both C and P.

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[1] I highly recommend checking your solution in case you get stuff backwards like I usually do.

3. Consider the set V consisting of cubic polynomials $p(x)$ defined on $[-1, 1]$ for which $p(-1) = p(1)$.

5 pts. (a) Show that this is a subspace of the vector space $\mathcal{P}_3[-1, 1]$ consisting of cubic polynomials defined on $[-1, 1]$.

8 pts. (b) Find a basis for V. (You need to show it is a basis).

10 pts. (c) Put the inner product

$$\langle p(x), q(x)\rangle = \int_{-1}^{1} p(x)q(x)dx$$

on V, and find a non-zero polynomial $r(x)$ in V which is orthogonal to the span of the set of polynomials $\{1, x^2\}$. (You must show your answer is indeed orthogonal to this space).

4. Let $T(x, y, z)$ be the linear transformation such that

$$T(1,0,0) = (1,1,1), \qquad T(1,0,1) = (1,1,2), \qquad T(1,1,1) = (3,3,5)$$

$\boxed{\text{5 pts.}}$ (a) Write the matrix corresponding to T.

$\boxed{\text{5 pts.}}$ (b) Give a basis for the image of T.

$\boxed{\text{5 pts.}}$ (c) Give a basis for the kernel of T.

$\boxed{\text{15 pts.}}$ 5. Find all real eigenvalues and corresponding eigenvectors for the linear transformation corresponding to the matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ -2 & 2 & -1 \\ 4 & 0 & 3 \end{pmatrix}$$