MATH 307 Second Midterm

November 10, 2010

Name: ______________________ **ID:** __________________

Question:	1	2	3	4	5	6	Total
Points:	15	15	15	15	15	15	90
Score:							

There are 6 problems on 5 pages in this exam (not counting the cover sheet). Make sure that you have them all.

You may use a calculator if you wish, provided your calculator does not do calculus. However, it is unlikely to be of much help.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, extra papers, and discussions with friends are not permitted.** Not study, test hard, wat do?

You have about 79 minutes and 47 seconds to complete this exam.

When you complete this exam, if there is sufficient time it is strongly recommended that you go back and reexamine your work, both on this exam and in your life up until now, for any errors that you may have made.

1. Let $f(x,y) = \cos(x)\sin(x^2+y^2)$.

$\boxed{\text{5 pts.}}$ (a) Calculate the gradient of f when $x = 0$ and $y = \dfrac{\sqrt{\pi}}{2}$.

$\boxed{\text{5 pts.}}$ (b) Write the equation of the plane tangent to the surface $z = f(x,y)$ at the point $\left(0, \dfrac{\sqrt{\pi}}{2}, \dfrac{1}{\sqrt{2}}\right)$.

$\boxed{\text{5 pts.}}$ (c) A particle is moving along the curve $\gamma(t) = \left\langle \dfrac{\sqrt{\pi}}{2}\cos t, \dfrac{\sqrt{\pi}}{2}\sin t \right\rangle$. Find the rate of change of $f(x,y)$ along this curve when $t = \pi/2$.

$\boxed{\text{15 pts.}}$ 2. Let $f : \mathbb{R}^3 \to \mathbb{R}$ and $g : \mathbb{R}^2 \to \mathbb{R}^3$ be given by $f(x,y,z) = x^2+y^2+z^2$, $\quad g(u,v) = \begin{pmatrix} u+v \\ u^3-v \\ u-v^3 \end{pmatrix}$.

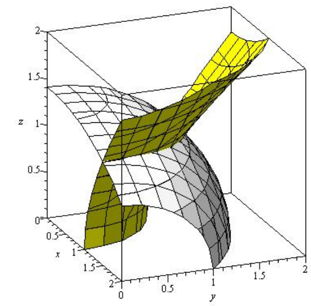
Write the derivative matrix of $f \circ g$ at a point (u,v).

$\boxed{\text{15 pts.}}$ 3. Let $f(x,y) = \begin{cases} \dfrac{x^3-y^3}{x^2+y^2} & \text{if } x^2+y^2 \neq 0 \\ 0 & \text{if } x^2+y^2 = 0 \end{cases}$. Is $f(x,y)$ continuous at all $(x,y) \in \mathbb{R}^2$?

If not, identify any discontinuities. Justify your answer fully.

$\boxed{\text{15 pts.}}$ 4. Two surfaces are given by $f(x, y, z) = 0$ and $g(x, y, z) = 0$, where

$$\begin{aligned} f(x, y, z) &= x^2 + 2y^2 + 3z^2 - 6 \\ g(x, y, z) &= x^2 + y^2 - z^2 - 1 \end{aligned}.$$



Let $\gamma(t)$ be the curve where they intersect.
Determine the line tangent to γ at $(1, 1, 1)$.

(Note: it is not necessary to determine $\gamma(t)$, but you may.)

$\boxed{\text{15 pts.}}$ 5. Find the point on the sphere $x^2 + y^2 + z^2 = 1$ which is furthest from the point $(1, 2, 3)$.

$\boxed{\text{15 pts.}}$ 6. Find all of the critical points of $x^3 - 6xy - 6y^2$. For each, state whether it is a local minimum, local maximum, or neither.