MAT 307: Multivariable Calculus with Linear Algebra
Topics for first exam, Fall 2010

The exam will cover everything we have done so far: chapters 1, 2, and 4 of the text.

The exam will be about an hour long. You should be familiar with all of the following topics:

Vectors and Coordinates: Be able to do simple arithmetic with vectors (adding them, etc.), finding their length, and so on. Be able to determine when a set of vectors are linearly independent.

The dot product: Definition ($A \cdot B = |A||B| \cos\theta$), calculation. Finding angles between vectors, computing a projection ($proj_A B$), etc. Know that $A \cdot B = 0$ if and only if A and B are perpendicular. Be aware that $A \cdot A = |A|^2$.

The cross product: Definition ($A \times B$ is perpendicular to both A and B, direction given by the right-hand rule, with $|A \times B| = |A||B| \sin\theta$). Computation of $A \times B$ as a determinant. The fact that $A \times B = -B \times A$, and that $A \times B = 0$ if and only if A and B are parallel.

Lines: If $P = (x_0, y_0, z_0)$ is a point on the line ℓ, and $V =< a, b, c >$ is a vector parallel to ℓ, the line an be written in vector form as $\ell(t) = P + tV$, or equivalently in parametric form as

$$x(t) = x_0 + at \quad y(t) = y_0 + bt \quad z(t) = z_0 + ct,$$

where the point P is (x_0, y_0, z_0) and the vector V is $\langle a, b, c\rangle$.

You should be able to work with equations of lines. For example, write the equation of a line given a point on it and a vector parallel to it, given two points on it, etc. Furthermore, you should be able to determine if two lines intersect, and if so, find the point of intersection and calculate the angle between the lines. If they don't intersect, you should be able to calculate the distance between them (this is harder than the others).

Planes: A plane can be described by giving a point on it and a vector perpendicular to it (a normal vector). If $P = (x_0, y_0, z_0)$ is a point on the plane, and $N =< a, b, c >$ is a vector normal to it, then the plane can be described as all points $R =< x, y, z >$ for which $N \cdot (R - P) = 0$, or equvalently $a(x - x_0) + b(y - y_0) + c(y - y_0) = 0$. Any equation of the form $ax + by + cz = d$ is a plane with normal vector $< a, b, c >$.

Planes may also be described parametrically by giving two vectors V and W spanning the plane's direction, and a point P; in this case, the plane has vector equation $P + tV + sW$ (as long as V and W are linearly independent). If $P = (x_0, y_0, z_0)$, $V = \langle v_1, v_2, v_3\rangle$ and $W = \langle w_1, w_2, w_3\rangle$, the vector-valued function giving the the plane will be $f(t, s) = \langle x_0 + tv_1 + sw_1, y_0 + tv_2 + sw_2, z_0 + sv_3 + sw_3\rangle$. This generalizes to any number of dimensions easily.

Be able to find the intersection of two planes, the distance from a point to a line, a point to a plane, angles between lines, planes, etc.

Matrices and vectors: Be able to do arithmetic with matrices and vectors (addition, subtraction, multiplication, scalar multiplication, etc.) Be able to solve a system of linear equations. Be able to find the inverse of a matrix.

Be able to calculate the determinant of a matrix, and know the relationship between the determinant and invertability of a matrix (determinant=0 if and only if the matrix is invertible). Know how row and column operations affect the value of the determinant.

Functions of one variable: Understand the description of a curve as a vector-valued function $R(t)$. Be able to differentiate and integrate vector functions. Computation of the tangent vector $R'(t)$ and the equation of the line tangent to the curve $R(t)$ at some point.

Functions of several values: Be able to identify the graph of a real-valued function of two variables, viewed as a surface in $\mathbb{R}^3$, as well as understand the level sets. Be able to calculate the partial derivatives of such a function. Understand parametric surfaces, and the vector partial derivative.