

# MATH 200

# Second Midterm

April 26, 2010

Name: \_\_\_\_\_ ID: \_\_\_\_\_

Question:	1	2	3	4	5	6	Total
Points:	10	10	10	10	10	5	55
Score:							

There are 6 problems in this exam. The pages are printed on both sides. Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, extra papers, and discussions with friends are not permitted.** You may contact the psychic friends network telepathically for help, but I don't think Miss Cleo or Dionne Warwick know much math.

You have about 80 minutes to complete this exam.

1. 10 points Suppose  $a$  is a nonzero rational number, and  $x$  is an irrational number. Prove that  $ax$  is an irrational number.

2. 10 points Let  $A$  be a set of positive integers with no least element. Show that  $A$  must be the empty set.

**Hint:** Prove by induction on  $n$  that if  $A$  has no least element, then  $\mathbb{N}_n$  is disjoint from  $A$  for every  $n$ .

3. (a) 5 points Carefully prove that if  $A$  and  $B$  are disjoint denumerable sets, then  $A \cup B$  is also denumerable.

(b) 5 points Show that if  $X$  is an uncountable set and  $A \subseteq X$  is denumerable, then the complement of  $A$  in  $X$  (that is,  $X - A$ ) must be uncountable.  
You may use the first part of this question, even if you couldn't do it

4. Three people decide to get tacos, and the tacqueria serves five kinds of tacos: beef, chicken, pork, fish, and vegetarian. Each person orders exactly one taco.
- (a) 5 points How many choices are possible if we record who selected which dish (as the waiter should)?

- (b) 5 points How many choices are possible if we forget who ordered which dish (as the chef might)?  
Be careful, this is more complicated than it may seem at first.

5. (a) 7 points Let  $X$  and  $Y$  be finite sets of the same cardinality. Prove that any surjective function  $f : X \rightarrow Y$  is also an injection.

(b) 3 points Suppose that  $X$  and  $Y$  are infinite sets of the same cardinality. Is it still true that any surjective function  $f : X \rightarrow Y$  is also injective? Prove or give a counterexample.

6. 5 points What is the coefficient of  $x^9$  in the expansion of  $(x + 2)^{12}$ ?