MATH 141 First Midterm

October 10, 2012

Name: ______________________ **ID:** __________________

Question:	1	2	3	4	Total
Points:	32	20	35	20	107
Score:					

There are 4 problems on 6 pages in this exam (not counting the cover sheet, but counting the blank page at the end). Make sure that you have them all.

You may use a calculator if you wish, provided your calculator does not do calculus. However, it is unlikely to be of much help.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, extra papers, and discussions with friends are not permitted.** Psychic consultation with deceased mathematicians is allowed, provided that you acknowledge them appropriately in your solution. Failure to do so constitutes academic dishonesty and will be prosecuted in accordance with university regulations.

You have a period of time which always seems too short to complete this exam. That period of time is not less than 50 minutes, but not much more, either.

When you complete this exam, if there is sufficient time it is strongly recommended that you go back and reexamine your work, both on this exam and in your life up until now, for any errors that you may have made.

$\boxed{\text{8 pts.}}$ 1. (a) Give a complete and careful definition of the statement "the sequence $\{a_k\}_{k=0}^{\infty}$ is bounded."

$\boxed{\text{8 pts.}}$ (b) Give a complete and careful definition of the following statement: "The sequence of real numbers $\{a_n\}_{n=1}^{\infty}$ converges to the limit L."

$\boxed{\text{8 pts.}}$ (c) State the Completeness Axiom for $\mathbb{R}$.

$\boxed{\text{8 pts.}}$ (d) Give a complete and careful definition of the statement "the number L is an accumulation point of the sequence $\{x_n\}_{n=0}^{\infty}$."

$\boxed{\text{20 pts.}}$ 2. Show that the function $g(x) = \begin{cases} x^2 & \text{for } x > 0 \\ -x & \text{for } x \leq 0 \end{cases}$ is continuous for all real numbers x.

3. Consider the sequence $\{a_n\}_{n=1}^{\infty}$ defined recursively by $a_1 = 1, \quad a_{n+1} = \frac{1}{2}\left(a_n + \frac{2}{a_n}\right).$

$\boxed{\text{15 pts.}}$ (a) Use induction to show that for all n, we have $1 \leq a_n \leq 2$

For the sequence given by $a_1 = 1, \quad a_{n+1} = \dfrac{1}{2}\left(a_n + \dfrac{2}{a_n}\right)$:

$\boxed{\text{15 pts.}}$ (b) Show that for $n > 2$, the sequence $\{a_n\}$ is decreasing. (Hint: look at a_{n+1}/a_n.)

$\boxed{\text{5 pts.}}$ (c) Does the sequence converge? Justify your answer.

4. For each of series below, determine if it converges. If it converges, give the limit and a brief justification. If it fails to converge, write that it diverges and give a justification.

10 pts. (a) $\displaystyle\sum_{j=1}^{\infty} \frac{\pi^{j+2}}{5^j}$

10 pts. (b) $\displaystyle\sum_{k=0}^{\infty} \frac{-k^3}{k^4 - \pi^4}$

This page was once a tree, and once it was blank. Now it is neither. You can make it less blank if you like. Turning it back into a tree is more effort, but also more worthwhile.