MATH 141 Second Midterm

November 17, 2010

Name: ______________________ **ID:** ____________________

Question:	1	2	3	4	5	6	Total
Points:	20	20	20	20	20	20	120
Score:							

There are 6 problems on 6 pages in this exam (not counting this cover sheet). Make sure that you have them all.

Do ANY 5 problems. Cross out the one you don't want graded.

You may use a calculator if you wish, provided your calculator does not do calculus. However, it is unlikely to be of much help.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, extra papers, and discussions with friends are not permitted.** Feel free to use psychic powers to read my mind for solutions to any of the problems. In the spirit of L'Hôpital, you may also purchase solutions to problems from Bernoulli (or Michelle).

When you complete this exam, if there is sufficient time it is strongly recommended that you go back and reexamine your work, both on this exam and in your life up until now, for any errors that you may have made.

5 pts. 1. (a) Give a complete and careful definition of the derivative of a function $f(x)$ at the point $x = a$.

5 pts. (b) Give a complete statement of the Mean Value Theorem.

5 pts. (c) Let $f : A \to B$ where A and B are both sets of real numbers. Define what the statement "The function g is the inverse of f" means.

5 pts. (d) Give a definition of the following statement: "The function $f(x)$ has a removable discontinuity at $x = a$."

20 pts. 2. For every integer $n \geq 2$, suppose that $g_n(x)$ is the product of n differentiable functions $f_1(x), f_2(x), \ldots, f_n(x)$. Prove that if $g_n(x) \neq 0$, then

$$g_n'(x) = g_n(x) \sum_{k=1}^{n} \frac{f_k'(x)}{f_k(x)}$$

You might find induction helpful.

20 pts. 3. Suppose that f is a continuous function from the interval $[0,1]$ with the property that $0 \leq f(x) \leq 1$ for every $x \in [0,1]$. Show that there is at least one number $c \in [0,1]$ such that $f(c) = c$. (Hint: consider the function $g(x) = f(x) - x$.)

20 pts. 4. Let

$$f(x) = \begin{cases} |x|^x & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

Show that $f(x)$ is a continuous function for all $x \in \mathbb{R}$.

5. For each of the functions below, calculate its derivative.

5 pts. (a) $e^{2x} \sec 3x$

5 pts. (b) $\arctan \sqrt{x-1}$

5 pts. (c) $\dfrac{(x^5 + 2x^3 + 8)\sqrt{x + x^2}}{(\sin 2x + \cos 2x)(x^4 - x^2 + 5)}$

5 pts. (d) $\ln(\ln(\ln(x)))$

20 pts.

6. The area between two varying concentric circle is 9π cm^2 at all times. The rate of change of the larger circle is 10π cm^2/sec. How fast is the circumference of the smaller circle changing when its area is 16π cm^2?