There are 9 problems in this exam, printed on 5 pages (not including this cover sheet). Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate clearly what is where if you expect someone to look at it. Books, calculators, electronic devices, extra papers, and discussions with friends are not permitted. Leave all answers in exact form (that is, do not approximate \( \pi \), square roots, and so on.) Telepathic communication with mathematically talented ducks is allowed, although you must also turn in the duck for grading along with your exam (geese, chickens, and other fowl are not permitted).

Use non-erasable pen (not red) if you want to be able to contest the grading of any problems. Questions with erasures will not be regraded.

You must give a correct justification of all answers to receive credit.

You have 90 minutes to complete this exam.
In questions 1 through 6, evaluate the given definite or indefinite integrals. If the integral does not converge, write **Divergent**. Don’t forget the constant of integration where relevant.

20 pts 1. \[ \int \frac{\ln(3t)}{t} \, dt \]

20 pts 2. \[ \int_{1}^{2} x^3 \ln x \, dx \]
In questions 1 through 6, evaluate the given definite or indefinite integrals. If the integral does not converge, write \textbf{Divergent}. Don’t forget the constant of integration where relevant.

\begin{enumerate}
\item \[ \int \frac{x - 2}{x^3 + x} \, dx \]
\item \[ \int_{1}^{6} \frac{dr}{(3 - r)^2} \]
\end{enumerate}
In questions 1 through 6, evaluate the given definite or indefinite integrals. If the integral does not converge, write **Divergent**. Don’t forget the constant of integration where relevant.

5. 20 pts  \[ \int \frac{y + 2}{\sqrt{4 - y^2}} \, dy \]

6. 20 pts  \[ \int e^{4x} \cos(x) \, dx \]
7. Find the area between the curves

\[ y = \frac{x}{2} \quad \text{and} \quad y^2 = 15 - x. \]

8. Does the improper integral

\[ \int_{2}^{\infty} \frac{x^{1/2} - 1}{5x^2 + 8} \, dx \]

converge? Fully justify your answer (note that if the integral converges, you need not give its value).
9. Since \( \int_{1}^{2} \frac{1}{x} \, dx = \ln 2 \), we can approximate \( \ln 2 \) using only addition, multiplication, and division by approximating the integral numerically.

**10 pts**  
(a) Use Simpson’s rule with 1 interval \((n = 2)\) to estimate \( \ln 2 = \int_{1}^{2} \frac{1}{x} \, dx \). You don’t have to add up the fractions.

**10 pts**  
(b) What \( n \) do we need to estimate \( \int_{1}^{2} \frac{1}{x} \, dx = \ln 2 \) within \( \frac{1}{1000} \) using Simpson’s rule?\(^1\)

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\(^1\)Use the following estimate for \( E_S \): If \( |f^{(4)}(x)| \leq K \) then \( E_S \leq K \frac{(b-a)^5}{180n^4} \).

It might be useful to know that \( 2^4 = 16, 3^4 = 81, 4^4 = 256, 5^4 = 625, 6^4 = 1296, 7^4 = 2401, 8^4 = 4096, 9^4 = 6561, \) and \( 10^4 = 10000 \). Or not.