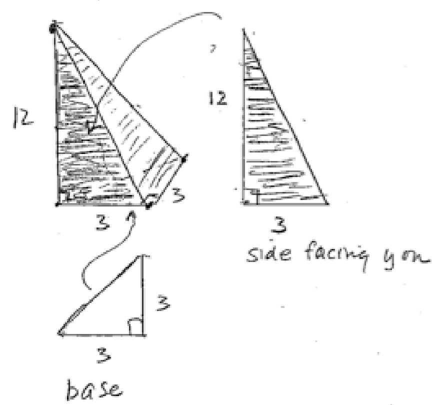
No calculators will be permitted and no tables provided. You should know the sines and cosines of the common angles $\pi/6, \pi/4, \pi/3, \pi/2, \pi$ and how to find the sines and cosines of angles for which these are reference angles. You should also be able to integrate $\sin^2(x)$ and $\cos^2(x)$.

2.2 The base of the solid pictured below is an isosceles right triangle, all cross sections parallel to the base are also isosceles right triangles, and the darkly shaded side facing you is a right triangle with side lengths as indicated. Find the volume of this sold.



2.3 Suppose a solid object has a flat horizontal base consisting of the region between the graphs of $y = x^2$ and $y = 1$ in the xy plane. The vertical cross sections perpendicular to the y axis are squares. Find the volume of the solid.

2.4 The region in the plane bounded by the y-axis, the graph of $y = \sin x$ and the line $y = 1$ is rotated around the axis $y = 1$. Find the volume of the resulting solid.

2.5 The graph of $y = \sqrt{1 - x^2/9}$ is the upper half of an ellipse. Find the volume of the solid obtained by rotating the region between this graph and the x-axis around the x-axis.

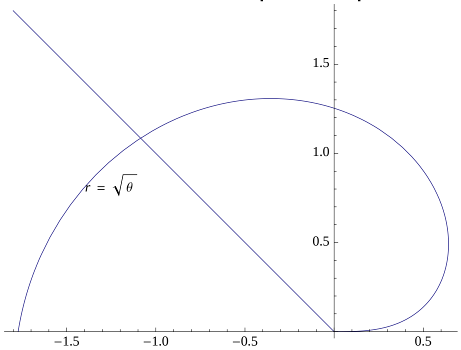
2.8 A cylindrical tank is 7 ft high and has radius 2 feet and stands on its circular base. The tank is initially filled to a depth of 5 ft with a liquid weighing 50 pounds per cubic foot. How much work is required to pump all the liquid in the tank to a point at the top of the tank? Express your answer as a definite integral. You don't need to evaluate it.

2.9 A 20 foot chain weighing 5 lbs/ft hangs vertically from the side of ship. Find the work required to lift the chain so that only a 3 foot length is left hanging.

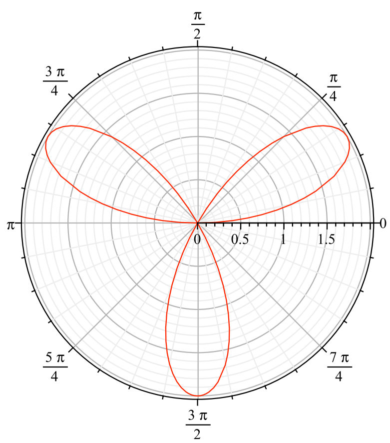
2.10 A bucket of sand weighing 1000 pounds initially is lifted by a crane at rate 1 foot/second to a height of 30 feet. As it is lifted, sand leaks out at the rate of 20 lbs/sec. Find the total work done in lifting the bucket of sand.

2.12 Find the length of the arc along the graph of $y = \frac{2}{3}x^{\frac{3}{2}}$ between the points $(0, 0)$ and $(3, 2\sqrt{3})$.

2.15 Find the area of the region between the horizontal axis, the line with equation $y = -x$ and the curve with polar equation $r = \sqrt{\theta}$.



2.16 a) Find the area enclosed by the part of the graph of $r = 2\sin(3\theta)$ contained in the first quadrant, i.e. the area of one of the three lobes in the graph shown.



c) Express as a definite integral the area of the part of the region in a) that lies outside the circle of radius 1 centered at the origin. Do not evaluate the integral.

2.17 Find the limits of each of the following sequences as $n \to \infty$ or say that the limit does not exist.

a) $s_n = \dfrac{4n^2 + 3n}{(1+n)(1+2n)}$

b) $s_n = 3 + \dfrac{(-1)^n}{\sqrt{n}}$

c) $s_n = \dfrac{n^2}{2^n}$

2.18 a) Find the sum of the infinite series

$$3 - 3/2 + 3/4 - 3/8 + 3/16 - 3/32 + 3/64 - 3/128 + \cdots$$

.

b) Find an explicit formula in terms of a for the sum

$$\sum_{n=1}^{21} 3a^{2n}.$$

No calculators will be permitted at the exam.

3.1 A ping-pong ball is launched straight up, rises to a height of 15 feet, then falls back to the launch point and bounces straight up again. It continues to bounce, each time reaching a height 90% of the height reached on the previous bounce. Find the total distance that the ball travels.

Note: Questions about convergence or divergence of series may be posed in various ways – some purely multiple choice, others requiring a coherent (specific or nonspecific) reason, some requiring choice and supporting detail. *Read questions carefully!*

3.2 Use the **integral test** to determine the convergence of the following series:

a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{e^n}$

3.3 Determine if the following series converge or diverge. Give your reasoning using complete sentences.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{(n+2)!}$

3.4 For each of the following items a) and b) choose a correct conclusion and reason from among the choices (R), (C), (I) below and provide supporting computation. For example, if you choose (R) calculate and interpret a suitable ratio.

(R) Converges by the ratio test. (C) Diverges by a p-series comparison.

(I) Converges by the integral test.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 3^n}{n 4^n}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$

3.5 a) Which of the following correctly classifies the series $\displaystyle\sum_{n=1}^{\infty} \frac{3^n + 4^n}{e^n}$ and gives a valid reason? Circle your answer. No other reason required.

i) Diverges: $\lim_{n\to\infty} a_n \neq 0$.

ii) Converges: sum of two convergent geometric series.

iii) Converges: Comparison test with geometric series with ratio $3/e$.

iv) Converges: Comparison test with geometric series with ratio $e/4$.

v) Diverges: constant multiple of the harmonic series.

b) Which of the following correctly classifies the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{5 + n^4}$ and gives a valid reason? Circle your answer. No other reason required.

i) Diverges: $\lim_{n\to\infty} a_n \neq 0$.

ii) Converges: Ratio test.

iii) Converges: Comparison test with a p-series, $p > 1$.

iv) Diverges: Ratio test.

v) Diverges: Comparison or limit comparison with the harmonic series.

3.6 Determine if the following series converge or diverge. Give your reasoning using complete sentences.

a) $\sum_{n=1}^{\infty}(-1)^n \frac{n}{n+2}$

b) $\sum_{n=2}^{\infty}(-1)^{n+1} \frac{1}{\ln n}$

3.7 a) Find the *interval* of convergence for the power series $\sum_{n=1}^{\infty} \frac{1}{n2^n}(x-3)^n$

b) Find the *radius* of convergence for $\sum_{n=1}^{\infty} \frac{n!}{(2n)!}x^n$