MAT132, Paper Homework 7

1. The hyperbolic sine function \( \sinh(x) \) and the hyperbolic cosine \( \cosh(x) \) are related as follows:

\[
\frac{d}{dx} \sinh(x) = \cosh(x) \quad \frac{d}{dx} \cosh(x) = \sinh(x)
\]
\[
\sinh(0) = 0 \quad \cosh(0) = 1
\]

Use the relationship above to determine the Maclaurin series for \( \sinh(x) \) and \( \cosh(x) \).

Hint: If \( f(x) = \cosh(x) \), the above facts tell you the value of all the derivatives \( f^{(n)}(0) \).

2. Using the series for \( \cosh(x) \) and \( \sinh(x) \) and the fact that \( i^2 = -1 \), verify that

- \( \cosh(x) = \frac{1}{2} (e^x + e^{-x}) \)
- \( \sinh(x) = \frac{1}{2} (e^x - e^{-x}) \)
- \( \cosh(x) = \cos(ix) \)
- \( \sinh(x) = -i \sin(ix) \)