MAT132, Paper Homework 10

1. There is considerable evidence to support the theory that for some species there is a minimum population m such that the species will become extinct if the size of the population falls below m. This condition can be incorporated into the logistic equation by introducing the factor $(1 - m/P)$. Thus the modified logistic model is given by the differential equation

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)\left(1 - \frac{m}{P}\right) \tag{1}$$

   a) Use the differential equation to show that any solution is increasing if $m < P < M$ and decreasing if $0 < P < m$.

   b) For the case $k = 0.08$, $M = 1000$, and $m = 200$, draw a direction field and use it to sketch several solution curves. Describe what happens to the population for various initial populations. What are the equilibrium solutions?

2. Using the ODE of equation (1):

   a) Algebraically solve the differential equation. Use initial population P_0.

   b) Use the solution you obtained in 2a) to show that if $P_0 < m$, then the species will become extinct.
   Hint: Show that the numerator in your expression for $P(t)$ is 0
   for some value of t.