1. A lemur rancher needs to invest in some high-tech lemur grooming machines. She determines that the machines will depreciate at a rate \( f(t) \), and the cost of keeping them in top running condition is given by another function \( g(t) \), where \( t \) is the time that the machines have been running.

The cost of keeping the machines around (instead of replacing them with new ones) is given by

\[
C(t) = \frac{1}{t} \int_0^t (f(t) + g(t)) \, dt
\]

Show the critical points of \( C(t) \) occur when \( C(t) = f(t) + g(t) \).

2. In the problem below, the identities \( \cos(\frac{\pi}{2} - x) = \sin(x) \) and \( \sin^2(x) + \cos^2(x) = 1 \) will be useful.

   (a) Use substitution to show that for any continuous function \( f \),
   \[
   \int_0^{\pi/2} f(\sin x) \, dx = \int_0^{\pi/2} f(\cos x) \, dx.
   \]

   (b) Using part (a) and the other trig identity, calculate
   \[
   \int_0^{\pi/2} \sin^2(x) \, dx \quad \text{and} \quad \int_0^{\pi/2} \cos^2(x) \, dx.
   \]