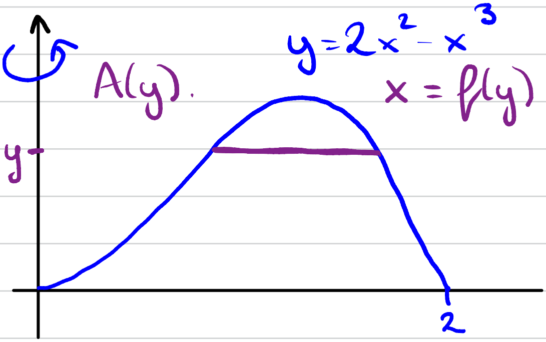
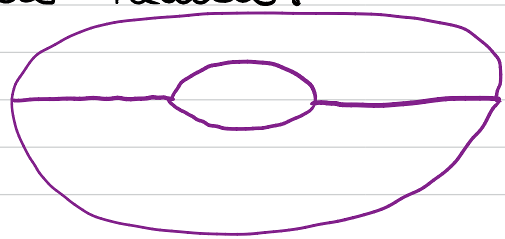
Topic: Volume by Shells

Some volumes are difficult to calculate using the disk method.

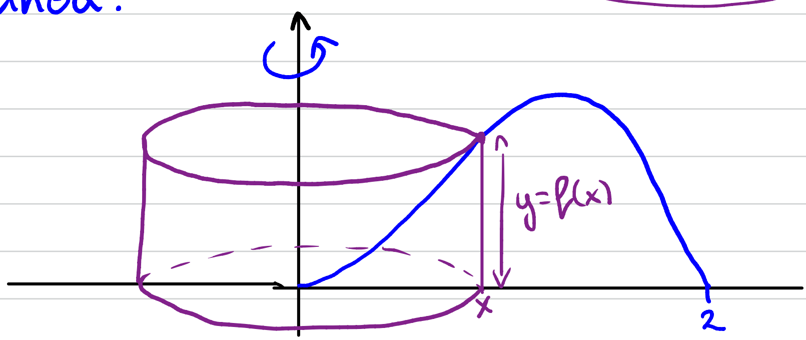
For example: Find volume obtained by rotating about the y-axis the region bounded by $y = 2x^2 - x^3$ and $y = 0$.



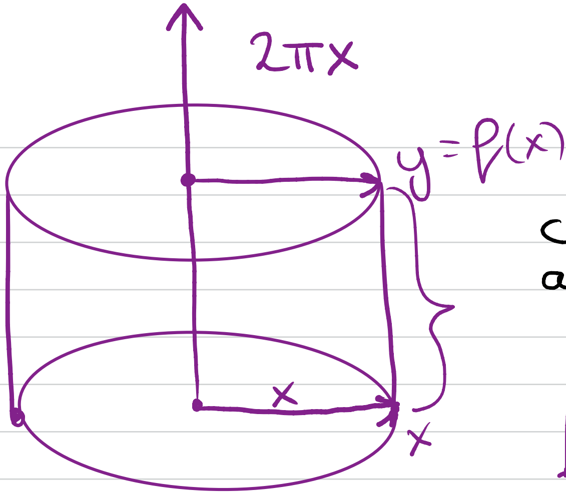
We could try the washer method, but it is very hard to compute inner and outer radius.



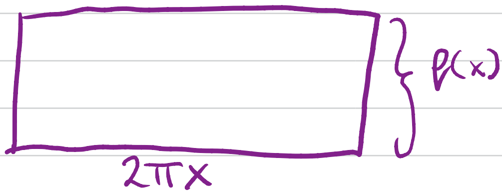
Instead, we will use the shell method:
What's the idea?



- cylindrical shell = surface area of a cylinder.



We get a cylindrical shell at each point.



Idea: calculate surface area of each shell, and add them up!

Area of the cylinder shell:

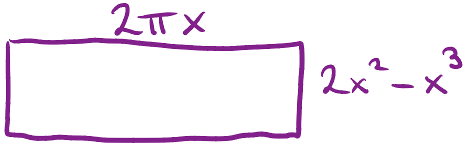
$$A(x) = 2\pi x f(x).$$

Just like previous method,

$$V = \int_a^b A(x)\,dx$$

surface area of each cylindrical shell

$$= \int_a^b 2\pi x f(x)\,dx.$$

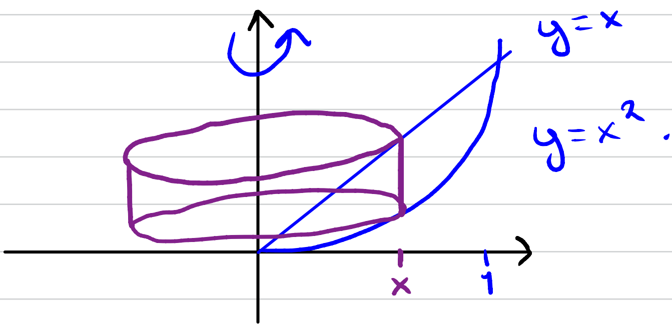


Let's do the example : $y = 2x^2 - x^3$

$$A(x) = 2\pi x(2x^2 - x^3)$$
$$= 2\pi(2x^3 - x^4).$$

$$V = \int_0^2 A(x)\,dx$$
$$= 2\pi \int_0^2 (2x^3 - x^4)\,dx$$
$$= 2\pi \left[\frac{x^4}{2} - \frac{x^5}{5} \right]_0^2 = \frac{16\pi}{5}.$$

<u>Example</u>: Find the volume of the solid obtained by rotating about the y-axis the region between $y = x$ and $y = x^2$.



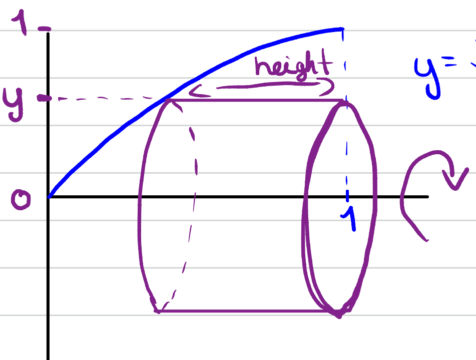
height $= x - x^2$.

$$A(x) = (x - x^2)\, 2\pi x$$
$$= 2\pi(x^2 - x^3).$$

$$V = \int_0^1 2\pi(x^2 - x^3)\,dx$$

$$= 2\pi\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = \frac{\pi}{6}.$$

Example: Find the Volume obtained by rotating the region under the curve $y = \sqrt{x}$ from 0 to 1 about the x-axis.



$y = \sqrt{x} \Rightarrow x = y^2.$

$\text{height} = 1 - y^2$

$$A(y) = 2\pi y(1 - y^2)$$

$$V(y) = \int_0^1 2\pi(y - y^3)\,dy = \frac{\pi}{2}.$$