Topic: Applications to Physics & Engineering

Work

Informally, work is the total amount of effort required to complete a task.

Technically: depends on the idea of a force

- If an object moves in a straight line with position function $s(t)$, force is defined:

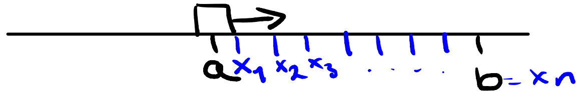
$$F = m\frac{d^2s}{dt^2} \qquad N = kg\,m/s^2$$

- If acceleration is constant, so is the force F.
- We define work done:

$$W = \text{force} \times \text{distance} = Fd$$

$$J = N \cdot m$$

- What happens if the Force F is changing?



Suppose object moves along x-axis from $x = a$ to $x = b$, and the force is given by

$$f(x).$$

- divide $[a, b]$ into n intervals, of width Δx
- choose sample pts $x_1^*, \ldots, x_n^*$.
- If n is very large, Δx is very small.
- $f(x_i^*) =$ force doesn't change much in $[x_i^*, x_{i+1}]$.

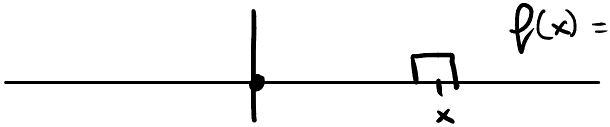
$$W \approx \sum_{i=1}^{n} \underbrace{f(x_i^*)}_{\text{force}} \times \underbrace{\Delta x}_{\text{distance}}.$$

We define the total work done:

$$W = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*) \Delta x$$

$$= \int_a^b f(x)\, dx$$

force at distance x.



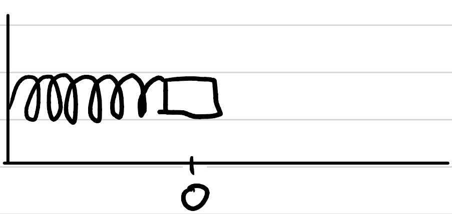
$f(x) = x^2 + 2x$

Example: When a particle is located a distance of x feet away from the origin, a force of $x^2 + 2x$ pounds acts on it.
How much work is done in moving it from $x=1$ to $x=3$?

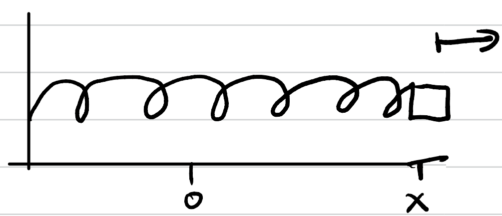
$$W = \int_1^3 f(x)\,dx$$
$$= \int_1^3 x^2 + 2x\,dx$$
$$= \frac{x^3}{3} + x^2 \Big|_1^3 = \frac{50}{3} \text{ lb.ft}$$

Example: A force of 40N is required to hold a spring that has been stretched from natural length of 10 cm to length of 15cm.
How much work is done stretching from 15cm to 18 cm?

Hooke's law: $f(x) = kx$



= 10cm.



k = spring constant.

We first find the spring constant k:
amount stretched $= 15-10$ cm $= 5$cm $= 0.05$ m
$k\,0.05 = f(0.05) = 40$
$k = 800$

Thus $f(x) = 800x$ N.

and $W = \int_{0.05}^{0.08} 800x \, dx$

$= 400x^2 \Big|_{0.05}^{0.08}$

$= 1.56$ J Nm