Today's Topic: Area between Curves

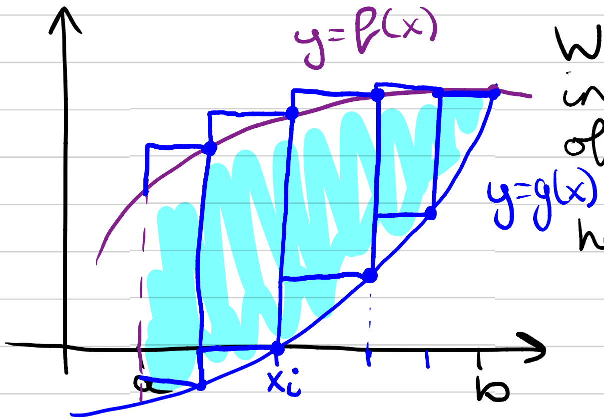
We know how to compute areas underneath a single curve.

What about regions enclosed by multiple curves?

Let $y = f(x)$, $y = g(x)$

where f, g are continuous

$f(x) \geq g(x)$ for all x in $[a, b]$.



We divide the region into n rectangles of equal width.

height - take right, left, or mid end point x_i.

height $= f(x_i) - g(x_i)$

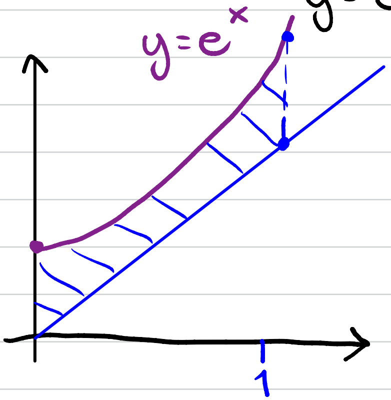
width $= \Delta x = \frac{b-a}{n}$

$$\text{Area} \approx \sum_{i=1}^{n} [f(x_i) - g(x_i)] \Delta x$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} (f(x_i) - g(x_i))\,\Delta x$$

$$A = \int_a^b [f(x) - g(x)]\,dx$$

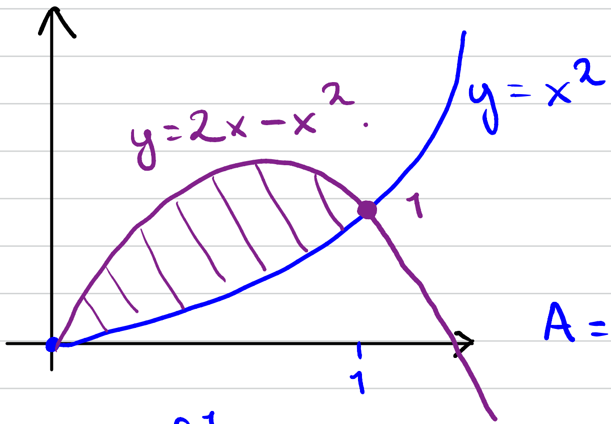
<u>Example</u> Find area enclosed by:
$y = e^x$; $y = x$ and $x = 0$, $x = 1$.



$$A = \int_0^1 [e^x - x]\,dx$$
$$= e^x - \frac{x^2}{2}\Big|_0^1$$
$$= e^1 - \frac{1}{2} - (e^0 - 0)$$
$$= e - 3/2.$$

<u>Example:</u> Find area enclosed by:
$y = x^2$; $y = 2x - x^2$.

Step 1: Find intersection points:
$$x^2 = 2x - x^2$$
$$2x^2 - 2x = 0$$
$$2x(x-1) = 0$$
$x = 0$ and $x = 1$.



$(1/2)^2 = 1/4$.

$2(1/2) - (1/2)^2 = 3/4$.

$2x - x^2 \geq x^2$ for all $x \in [0, 1]$.

$$A = \int_0^1 (2x - x^2) - x^2 \, dx$$

$$A = \int_0^1 2x - 2x^2 \, dx$$

$$= 2\int_0^1 x - x^2 \, dx$$

$$= 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$= 2\left[\frac{1}{2} - \frac{1}{3}\right] - 2\left[\frac{0^2}{2} - \frac{0^3}{3}\right]$$

$$= \frac{1}{3}.$$