(20) $\int x\cos(3x)\,dx$ $\quad u = x \quad dv = \cos(3x)\,dx$

$du = dx \quad v = \frac{\sin(3x)}{3}$

$$\int x\cos(3x)\,dx = \frac{x\sin(3x)}{3} - \frac{1}{3}\int \sin(3x)\,dx$$

$$= \frac{x\sin(3x)}{3} - \frac{1}{3}\left(\frac{-\cos(3x)}{3}\right) + C$$

$$= \frac{x\sin(3x)}{3} + \frac{1}{9}\cos(3x) + C$$

(21) $\int \cos^5 x \sin x\,dx$ $\quad$ Let $u = \cos x$

$du = -\sin x\,dx$

$$= -\int u^5\,du = -\frac{u^6}{6} + C = -\frac{\cos^6 x}{6} + C$$

(22) $\int \sin^4 3x \cos 3x \, dx$

$u = \sin 3x$

$du = \frac{\cos 3x}{3} dx$

$3du = \cos 3x \, dx$

$= 3\int u^4 du = \frac{3u^5}{5} = \frac{3\sin^5(3x)}{5} + C$

---

(18) (oops!) $\int x\sec^2 x \, dx$

$u = x$ $\quad dv = \sec^2 x \, dx$

$du = dx$ $\quad v = \tan x$

$$\int x\sec^2 x \, dx = x\tan x - \int \tan x \, dx$$

$$= x\tan x + \ln(\cos x) + C$$

---

(23) $\int \cos^2 4x \, dx = \frac{1}{2}\int (1+\cos 8x) dx = \frac{1}{2}\left(x + \frac{\sin 8x}{8}\right) + C$

---

(24) $\int \cos^3 x \, dx = \int \cos x \cos^2 x \, dx = \int \cos x (1-\sin^2 x) dx$

$u = \sin x$

$du = \cos x \, dx$

$$\int 1-u^2 du = u - \frac{u^3}{3} + C = \sin x - \frac{\sin^3 x}{3} + C$$

(25) $\int \sin^3 x \cos^3 x\,dx = \int \sin^3 x \cos^2 x \cos x\,dx$

$= \int \sin^3 x(1-\sin^2 x)\cos x\,dx$

$u = \sin x$
$du = \cos x\,dx$

$= \int u^3(1-u^2)\,du = \int u^3 - u^5\,du = \frac{u^4}{4} - \frac{u^6}{6} + C$

$= \frac{\sin^4 x}{4} - \frac{\sin^6 x}{6} + C$

---

(26) $\int \cos^2 x \sin^2 x\,dx = \int \frac{1}{2}(1+\cos 2x)\cdot\frac{1}{2}(1-\cos 2x)\,dx$

$= \frac{1}{4}\int (1-\cos^2 2x)\,dx = \frac{1}{4}\int \sin^2 2x\,dx = \frac{1}{4}\int \frac{1}{2}(1-\cos 4x)\,dx$

$= \frac{1}{8}\left(x - \frac{\sin 4x}{4}\right) + C$

---

(27) $\int \tan^2 x \sec^2 x\,dx$ $\quad u = \tan x$
$du = \sec^2 x\,dx$

$\int u^2\,du = \frac{u^3}{3} + C$

$= \frac{\tan^3 x}{3} + C$

(28) $$\int \tan^3 x \sec^4 x \, dx = \int \tan^3 x \sec^2 x \sec^2 x \, dx$$

$$= \int \tan^3 x \, (1+\tan^2 x) \sec^2 x \, dx$$

let $u = \tan x$

$du = \sec^2 x \, dx$

$$= \int u^3 (1+u^2) du = \int u^3 + u^5 du = \frac{u^4}{4} + \frac{u^6}{6} + C$$

$$= \frac{\tan^4 x}{4} + \frac{\tan^6 x}{6} + C$$

(29) $$\int \tan^5 x \sec x \, dx = \int (\tan^2 x)^2 (\sec x \tan x) \, dx$$

$$= \int (\sec^2 x - 1)^2 (\sec x \tan x) dx$$

$u = \sec x$

$du = \sec x \tan x \, dx$

$$= \int (u^2 - 1)^2 du = \int u^4 - 2u^2 + 1 \, du$$

$$= \frac{u^5}{5} - \frac{2u^3}{3} + u + C$$

$$= \frac{\sec^5 x}{5} - \frac{2\sec^3 x}{3} + \sec x + C$$

30) $\int \sec^3 x\, dx$ $\quad u = \sec x \quad dv = \sec^2 x\, dx$

$\qquad du = \sec x \tan x\, dx \quad v = \tan x$

$$\int \sec^3 x\, dx = \sec x \tan x - \int \sec x \tan^2 x\, dx$$

$$\int \sec^3 x\, dx = \sec x \tan x - \int \sec x\, (\sec^2 x - 1)\, dx$$

$$\int \sec^3 x\, dx = \sec x \tan x - \int \sec^3 x\, dx + \int \sec x\, dx$$

$$2\int \sec^3 x\, dx = \sec x \tan x + \int \sec x\, dx$$

$$2\int \sec^3 x\, dx = \sec x \tan x + \ln|\sec x + \tan x| + C$$

$$\int \sec^3 x\, dx = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C$$

(31) $\int \frac{dx}{x^2+3x-4} = \int \frac{dx}{(x+4)(x-1)}$

$$\frac{1}{(x+4)(x-1)} = \frac{A}{x+4} + \frac{B}{x-1}$$

$$1 = A(x-1) + B(x+4)$$
$$1 = Ax - A + Bx + 4B$$
$$1 = Ax + Bx + (-A+4B)$$

$$A + B = 0$$
$$-A + 4B = 1$$
$$5B = 1$$
$$B = \frac{1}{5} \to A = -\frac{1}{5}$$

$$\int \frac{dx}{(x+4)(x-1)} = -\frac{1}{5}\int \frac{dx}{x+4} + \frac{1}{5}\int \frac{dx}{x-1}$$

$$= -\frac{1}{5}\ln|x+4| + \frac{1}{5}\ln|x-1| + C$$

---

(32) $\int \frac{11x+17}{2x^2+7x-4}\,dx = \int \frac{11x+17}{(2x-1)(x+4)}\,dx$

$$\frac{11x-17}{(2x-1)(x+4)} = \frac{A}{2x-1} + \frac{B}{x+4} \to 11x-17 = A(x+4) + B(2x-1)$$

$$11x - 17 = Ax + 4A + 2Bx - B$$

$$A + 2B = 11 \to A + 2B = 11$$
$$4A - B = -17 \quad 8A - 2B = -34$$
$$9A = -23$$
$$A = -\frac{23}{9}$$
$$B = \frac{61}{9}$$

$$\int \frac{11x+17}{2x^2+7x-4}\,dx = -\frac{23}{9}\int \frac{dx}{2x-1} + \frac{61}{9}\int \frac{dx}{x+4}$$

$$= -\frac{23}{9}\,\frac{\ln|2x-1|}{2} + \frac{61}{9}\ln|x+4| + C$$

(33) $\displaystyle\int \frac{5x-5}{3x^2-8x-3}\,dx$ $\qquad \displaystyle\frac{5x-5}{3x^2-8x-3} = \frac{A}{3x+1} + \frac{B}{x-3}$

$(3x+1)(x-3)$

$$5x-5 = A(x-3)+B(3x+1)$$
$$5x-5 = Ax-3A+3Bx+B$$

$A+3B=5 \rightarrow 3A+9B=15$

$-3A+B=-5 \qquad -3A+3B=-5$

$12B=10$

$B=\frac{5}{6}$

$A=\frac{25}{18}$

$$\int \frac{5x-5}{3x^2-8x-3}\,dx = \frac{25}{18}\int \frac{dx}{3x+1} + \frac{5}{6}\int \frac{dx}{x-3}$$
$$= \frac{25}{18}\,\frac{\ln|3x+1|}{3} + \frac{5}{6}\ln|x-3| + C$$

(34) $\displaystyle\int \frac{2x^2-1}{(4x-1)(x^2+1)}\,dx$ $\qquad \displaystyle\frac{2x^2-1}{(4x-1)(x^2+1)} = \frac{A}{4x-1} + \frac{Bx+C}{x^2+1}$

$$2x^2-1 = Ax^2+A+4Bx^2+4Cx-Bx-C$$
$$2x^2-1 = Ax^2+4Bx^2+4Cx-Bx+A-C$$

$A+4B=2 \longrightarrow A+16C=2$

$4C-B=0 \rightarrow B=4C$

$A-C=-1 \longrightarrow A-C=-1$

$17C=3$

$C=\frac{3}{17} \quad B=\frac{12}{17} \quad A=\frac{-14}{17}$

$$\int \frac{2x^2-1}{(4x-1)(x^2+1)}\,dx = \frac{-14}{17}\int \frac{dx}{4x-1} + \frac{12}{17}\int \frac{x\,dx}{x^2+1} + \frac{3}{17}\int \frac{dx}{x^2+1}$$

$u=x^2+1$

$du=2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \frac{-14}{17}\,\frac{\ln|4x-1|}{4} \qquad \frac{6}{17}\int \frac{du}{u} = \frac{6}{17}\ln|u| \qquad +\frac{3}{17}\operatorname{Tan}^{-1}x + C$

$$= -\frac{14}{}\ln|4x-1| + \frac{6}{}\ln|x^2+1| + \frac{3}{}\operatorname{Tan}^{-1}x + C$$

35 $\int \frac{dx}{x(x^2+1)}$ $\qquad \frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

$1 = Ax^2 + A + Bx^2 + Cx$

$A+B=0$

$C=0$

$A=1$

$B=-1$

$$\int \frac{dx}{x(x^2+1)} = \int \frac{dx}{x} - \int \frac{x}{x^2+1}\,dx$$

$= \ln|x| -$ $\qquad u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$\frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| = \frac{1}{2}\ln|x^2+1|$$

$$= \ln|x| - \frac{1}{2}\ln|x^2+1| + C$$