① $\int (3x-1)^5 dx$ let $u = 3x-1$

$du = 3\,dx$

$\frac{1}{3} du = dx$

$= \frac{1}{3}\int u^5 du$

$= \frac{1}{3}\left(\frac{u^6}{6}\right) + C = \frac{u^6}{18} + C = \frac{(3x-1)^6}{18} + C$

---

② $\int x(2-x^2)^3 dx$ let $u = 2-x^2$

$du = -2x\,dx$

$-\frac{1}{2} du = x\,dx$

$= -\frac{1}{2}\int u^3 du$

$= -\frac{1}{2}\left(\frac{u^4}{4}\right) + C = -\frac{u^4}{8} + C = -\frac{(2-x^2)^4}{8} + C$

---

③ $\int \sin 3x\, dx$ let $u = 3x$

$du = 3\,dx$

$\frac{1}{3} du = dx$

$= \frac{1}{3}\int \sin u\, du$

$= -\frac{1}{3}\cos u + C = -\frac{1}{3}\cos(3x) + C$

---

④ $\int \sqrt{3x-1}\, dx$ let $u = 3x-1$

$du = 3\,dx$

$\frac{1}{3} du = dx$

$= \frac{1}{3}\int u^{\frac{1}{2}} du$

$= \frac{1}{3}\,\frac{u^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{9}u^{\frac{3}{2}} + C = \frac{2}{9}(3x-1)^{\frac{3}{2}} + C$

(5) $\int x\sqrt{7x^2+12}\,dx$ let $u = 7x^2+12$

$du = 14x\,dx$

$\frac{1}{14}du = x\,dx$

$= \frac{1}{14}\int u^{\frac{1}{2}}du$

$= \frac{1}{14}\,\frac{u^{\frac{3}{2}}}{\frac{3}{2}} = \frac{1}{21}u^{\frac{3}{2}} + C = \frac{1}{21}(7x^2+12)^{\frac{3}{2}} + C$

---

(6) $\int \frac{x}{(4x^2+1)^3}dx$ Let $u = 4x^2+1$

$du = 8x\,dx$

$\frac{1}{8}du = x\,dx$

$= \frac{1}{8}\int \frac{du}{u^3} = \frac{1}{8}\int u^{-3}du$

$= \frac{1}{8}\,\frac{u^{-2}}{-2} = -\frac{1}{16}\cdot\frac{1}{u^2} = -\frac{1}{16}\cdot\frac{1}{(4x^2+1)^2}\,dx$

---

(7) $\int \frac{\sin(\frac{5}{x})}{x^2}dx$ let $u = \frac{5}{x}$

$du = -\frac{5}{x^2}dx$

$-\frac{1}{5}du = \frac{dx}{x^2}$

$= -\frac{1}{5}\int \sin u\,du$

$= \frac{1}{5}\cos u + C = \frac{1}{5}\cos\left(\frac{5}{x}\right) + C$

8) $$\int \frac{\sec^2(\sqrt{x})}{\sqrt{x}}\,dx$$ let $u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2du = \frac{dx}{\sqrt{x}}$

$$= 2\int \sec^2 u\,du$$

$$= 2\tan u + C = 2\tan\sqrt{x} + C$$

---

9) $$\int \sin(\sin\theta)\cos\theta\,d\theta$$ let $u = \sin\theta$

$du = \cos\theta\,d\theta$

$$= \int \sin u\,du = -\cos u + C = -\cos(\sin\theta) + C$$

---

10) $$\int x\sqrt{x-3}\,dx$$ let $u = x-3$

$u+3 = x$

$du = dx$

$$= \int (u+3)\sqrt{u}\,du$$

$$= \int u^{\frac{3}{2}} + 3u^{\frac{1}{2}}\,du = \frac{u^{\frac{5}{2}}}{\frac{5}{2}} + \frac{3u^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= \frac{2}{5}u^{\frac{5}{2}} + 2u^{\frac{3}{2}} + C$$

$$= \frac{2}{5}(x-3)^{\frac{5}{2}} + 2(x-3)^{\frac{3}{2}} + C$$

(11) $\int xe^{-x}\,dx$ $\qquad u = x \quad dv = e^{-x}dx$

$\qquad du = dx \quad v = -e^{-x}$

$$\int xe^{-x}dx = -xe^{-x} + \int e^{-x}dx$$
$$= -xe^{-x} - e^{-x} + C$$

---

(12) $\int \ln(2x+3)\,dx$ $\qquad u = \ln(2x+3) \quad dv = dx$

$\qquad du = \frac{2}{2x+3}dx \quad v = x$

$$\int \ln(2x+3)dx = x\ln(2x+3) - \int \frac{2x}{2x+3}dx$$
$$= x\ln(2x+3) - \int\left(1 - \frac{3}{2x+3}\right)dx$$

Polynomial division

$$= x\ln(2x+3) - \left(x - \frac{3\ln(2x+3)}{2}\right) + C$$

---

(13) $\int x\ln x\,dx$ $\qquad u = \ln x \quad dv = x\,dx$

$\qquad du = \frac{1}{x}dx \quad v = \frac{x^2}{2}$

$$\int x\ln x\,dx = \frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx$$
$$= \frac{x^2}{2}\ln x - \frac{1}{2}\left(\frac{x^2}{2}\right) + C$$

(14) $\int \sin^{-1}x \, dx$

$u = \sin^{-1}x \qquad dv = dx$

$du = \frac{1}{\sqrt{1-x^2}}\,dx \qquad v = x$

$$\int \sin^{-1}x\,dx = x\sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}}\,dx$$

Let $u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$$= -\frac{1}{2}\int u^{-\frac{1}{2}}du = \frac{-u^{\frac{1}{2}}}{2\cdot\frac{1}{2}} = -u^{\frac{1}{2}}$$

$$= -\sqrt{1-x^2}$$

$$\int \sin^{-1}x\,dx = x\sin^{-1}x + \sqrt{1-x^2} + C$$

---

(15) $\int x^2 e^{-x}\,dx$

$u = x^2 \qquad dv = e^{-x}dx$

$du = 2x\,dx \qquad v = -e^{-x}$

$$\int x^2e^{-x}dx = -x^2e^{-x} + 2\int xe^{-x}dx$$

$u = x \quad dv = e^{-x}dx$

$du = dx \quad v = -e^{-x}$

$$= -x^2e^{-x} + 2\left(-xe^{-x} + \int e^{-x}dx\right)$$

$$= -x^2e^{-x} - 2xe^{-x} + 2\int e^{-x}dx$$

$$= -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C$$

(16) $\int e^x \sin x\,dx$ $\quad u = e^x \quad dv = \sin x\,dx$
$\quad du = e^x dx \quad v = -\cos x$

$$\int e^x \sin x\,dx = -e^x\cos x + \int e^x \cos x\,dx$$

$u = e^x \quad dv = \cos x\,dx$
$du = e^x dx \quad v = \sin x$

$$\int e^x \sin x\,dx = -e^x\cos x + e^x \sin x - \int e^x \sin x\,dx$$

add this integral to both sides

$$2\int e^x \sin x\,dx = -e^x \cos x + e^x \sin x + C$$

$$\int e^x \sin x\,dx = \frac{-e^x\cos x + e^x \sin x}{2} + C$$

---

(17) $\int x \tan^{-1} x\,dx$ $\quad u = \tan^{-1}x \quad dv = x\,dx$
$\quad du = \frac{1}{1+x^2}dx \quad v = \frac{x^2}{2}$

$$\int x\tan^{-1}x\,dx = \frac{x^2}{2}\tan^{-1}x + \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx$$

polynomial division

$$= \frac{x^2}{2}\tan^{-1}x + \frac{1}{2}\int 1 - \frac{1}{1+x^2}\,dx$$

$$= \frac{x^2}{2}\tan^{-1}x + \frac{1}{2}\left(x - \tan^{-1}x\right) + C$$

(19) $\int x^2 \ln x \, dx$ $\quad u = \ln x \quad dv = x^2 dx$

$\quad du = \frac{1}{x} dx \quad v = \frac{x^3}{3}$

$$\int x^2 \ln x \, dx = \frac{x^3}{3} \ln x - \frac{1}{3}\int x^2 \, dx$$

$$= \frac{x^3}{3} \ln x - \frac{1}{3} \cdot \frac{x^3}{3} + C$$

(20) $\int x\cos(3x)\,dx$ $\quad u = x \quad dv = \cos(3x)\,dx$

$\quad du = dx \quad v = \frac{\sin(3x)}{3}$

$$\int x\cos(3x)\,dx = \frac{x\sin(3x)}{3} - \frac{1}{3}\int \sin(3x)\,dx$$

$$= \frac{x\sin(3x)}{3} - \frac{1}{3}\left(\frac{-\cos(3x)}{3}\right) + C$$

$$= \frac{x\sin(3x)}{3} + \frac{1}{9}\cos(3x) + C$$