MAT 126 Calculus Spring 2006
Practice Final Exam

Answer each question in the space provided and on the reverse side of the sheets. Show your work whenever possible. Unless otherwise indicated, **answers without justification will get little or no partial credit!** Cross out anything that grader should ignore and circle or box the final answer. You **do not need** to simplify numerical answers or write their approximate values. This practice exam contains more problems then the actual test to give you more practice.

1. Evaluate the following definite integrals:

(a)
$$\int_1^9 \ln \sqrt{x} \, dx$$

(b)
$$\int_0^2 \frac{x}{1 + 2x^2} \, dx$$

(c)
$$\int_1^e \frac{(\ln x)^3}{x} \, dx$$

(d)
$$\int_{-1}^1 x^2 \sin(x^5) \, dx$$

(e)
$$\int_0^{1/2} \frac{\sin^{-1} x}{\sqrt{1 - x^2}} \, dx$$

(f)
$$\int_1^4 \sqrt{t} \ln t \, dt$$

(g)
$$\int_0^{13} \frac{dx}{\sqrt[3]{(1 + 2x)^2}}$$

(h)
$$\int_0^{\frac{\pi}{2}} \sin^3 x \, dx$$

2. Evaluate the following indefinite integrals:

(a)
$$\int x^2 e^x \, dx$$

(b)
$$\int \frac{2x^3 + 1}{x^2 + 1} \, dx$$

(c)
$$\int \frac{\tan^{-1} x}{1 + x^2} \, dx$$

(d)
$$\int \sin^{-1} x \, dx$$

(e)
$$\int \frac{x - 1}{x^2 + 3x + 2} \, dx$$

(f)
$$\int t^2 \cos(1 - t^3) \, dt$$

(g)
$$\int e^x \sqrt[3]{1 + e^x} \, dx$$

(h)
$$\int \cos^5 x \, dx$$

3. (a) Write a formula for $\cos^2 x$ in terms of $\sin^2 x$.
(b) Evaluate
$$\int \cos^3 x \sin^2 x \, dx$$

4. Let
$$f(x) = \int_2^{\sqrt{x}} \frac{\sin t}{t} \, dt + x^2$$

(a) Find $f'(x)$.
(b) Evaluate $f(4)$.

5. Find a function f and a number a such that for x,
$$1 + \int_a^x t f(t) \, dt = x^3$$

6. (a) Let
$$I = \int_0^4 e^{x^2} \, dx$$

For any value of n list the numbers L_n, R_n, M_n, T_n and I in increasing order.

(b) Repeat part (a) for

$$I = \int_0^{\sqrt{2}/2} e^{-x^2}\,dx$$

7. Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

(a)
$$\int_0^\infty e^{-x}\,dx$$

(b)
$$\int_0^1 \frac{1}{\sqrt{x}}\,dx$$

(c)
$$\int_0^3 \frac{1}{x\sqrt{x}}\,dx$$

(d)
$$\int_{-\infty}^\infty xe^{-x^2}\,dx$$

(e)
$$\int_0^1 \frac{1}{4y-1}\,dy$$

8. Find the area of the region bounded by the curves:
(a) $y = x^2$ and $y = x^4$.
(b) $x + y^2 = 2$ and $x + y = 0$.

9. (a) Find the volume of the solid of revolution obtained by rotating the region bounded by the curves $y = x^2$ and $y^2 = x$ about the x-axis.
(b) Find the volume of the solid of revolution obtained by rotating the region bounded by $y = \sec x$, $y = 1$, $x = -1$ and $x = 1$ about the x-axis.

10. Find the length of the following curves:
(a) $y = x^{3/2}$, $0 \le x \le 2$.
(b)
$$y = \frac{x^2}{4} - \frac{\ln x}{2}, \quad 1 \le x \le 2$$

11. Find the average value f_{ave} of f on the given interval.
(a) $f(x) = x\sin(x^2)$ on $[0, \sqrt{\pi}]$.
(b) $f(x) = 4 - x^2$ on $[0, 3]$.
(c) For f as in part (b) find the number c in $[0, 3]$ such that $f(c) = f_{ave}$.