

**Part 2:** These will be graded **only** if you have passed part 1. **Name:** \_\_\_\_\_

13. Find an antiderivative (that is, a function whose derivative is the given function) for each of the following functions:

3 points

(a)  $f(x) = 3x^3 - 6x^2 + x + e^2$

(a) \_\_\_\_\_

3 points

(b)  $g(x) = \sqrt{2+x}$

(b) \_\_\_\_\_

3 points

(c)  $h(x) = \frac{x}{5} - \frac{5}{x}$

(c) \_\_\_\_\_

3 points

(d)  $k(x) = \frac{2}{\sqrt{1-x^2}}$

(d) \_\_\_\_\_

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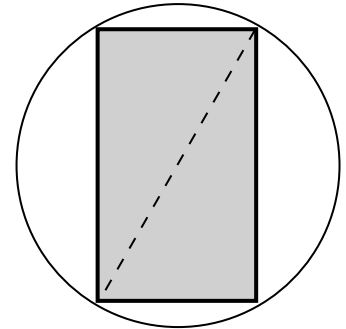
12 points

14. The strength of a rectangular beam of a given length is proportional to the width times the square of the height of a cross-section, that is,

$$S = w \cdot h^2$$

where  $S$  is the strength (in some appropriate units),  $w$  is the width, and  $h$  is the height.

Find the dimensions of the strongest beam that can be cut out of a log which has a circular cross-section with a 9" diameter (that is, the cross-section of the beam is a rectangle with a diagonal of 9").



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8 points

15. (a) Find  $\frac{dy}{dx}$  at the point  $(1, 1)$  if  $x$  and  $y$  satisfy  $\frac{5y^2 - 2x}{4y^3 - x^2} = x$ .

(Hint: you do not need to use the quotient rule if you do some algebra first.)

4 points

(b) Write the equation of the line tangent to the curve at the point  $(1, 1)$ .

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16. Consider the function  $f(x) = 3x^{\frac{2}{3}} - 2x$ .

4 points

(a) Find the  $x$ -values of all critical points of  $f(x)$

4 points

(b) State the largest interval on which  $f(x)$  is increasing.

4 points

(c) Give the  $x$ -values at which the absolute maximum and absolute minimum values of  $f$  occur when  $-1 \leq x \leq 3$ .

(You might find it helpful to know that  $2^{\frac{2}{3}} \approx 1.59$ ,  $3^{\frac{2}{3}} \approx 2.08$ , and  $4^{\frac{2}{3}} \approx 2.52$ .)

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17. Jimi Chiu makes “designer” shoes<sup>1</sup> that he sells at \$150 a pair. He knows that the number of pairs of shoes he sells is a function of the price he charges; let’s denote this by  $N(p)$ , where  $p$  is the price per pair. Market research tells him that  $N'(150)$  is about  $-10$ ; that is, if he raises the price by one dollar, he should expect to sell 10 fewer pairs. The amount of revenue  $R(p)$  he makes at a given price will be given by  $R(p) = p \cdot N(p)$ .

7 points

(a) If he typically sells 2000 pairs of shoes at \$150 each, what is  $R'(150)$ ?

5 points

(b) Use your answer above to estimate his revenue if he raises the price to \$160 per pair (that is, estimate  $R(160)$ ). Should he raise the price?

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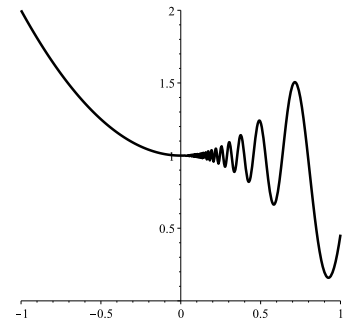
<sup>1</sup>No relation to Jimmy Choo shoes, unless you don’t look very closely. Mr. Chiu is also fond of Rollex watches.

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18. Consider the function

$$Q(x) = \begin{cases} 1 + x^2 & x \leq 0 \\ 1 + x^2 \sin(10/x) & \text{otherwise} \end{cases}$$

with the graph shown at right.



6 points

(a) Show that  $Q(x)$  is continuous at every value of  $x$ .

6 points

(b) Is  $Q(x)$  differentiable at  $x = 0$ ? If so, what is  $Q'(0)$ ? If not, why not?