Part 2: These will be graded **only** if you have passed part 1. **Name:** ____________

13. Find an antiderivative (that is, a function whose derivative is the given function) for each of the following functions:

3 points (a) $f(x) = 2x^3 - 6x^2 + 8x + e^2$

$\frac{1}{2}x^4 - 2x^3 + 4x^2 + e^2x + C$

(a) ____________

3 points (b) $g(x) = e^{2x} + \sin(3x)$

$\frac{1}{2}e^{2x} - \frac{1}{3}\cos(3x) + C$

(b) ____________

3 points (c) $h(x) = 5\sqrt{x} - \frac{3}{x}$

$\frac{15}{2}x^{3/2} - 3\ln|x| + C$

(c) ____________

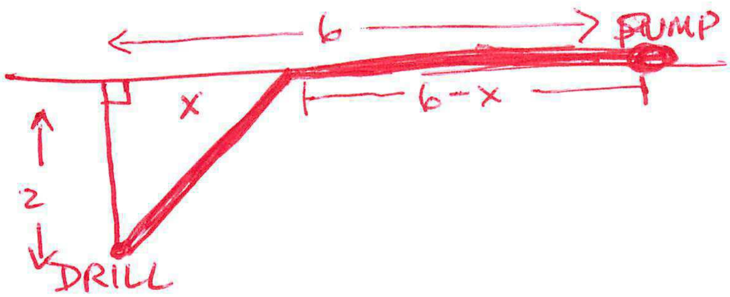
3 points (d) $k(x) = \frac{5}{1+x^2}$

$5\arctan x + C$

(d) ____________

Part 2: These will be graded **only** if you have passed part 1. **Name:** ____________

12 points | 14. A company plans to build a pipeline from its drilling station, which is located in the ocean 2 miles south from a straight shoreline running east-west, to a pumping station which is located 6 miles east from the point on the shore directly opposite the drilling station. The pipeline will cost \$600/mile to run under the water and \$200/mile to run under the land. Where should the pipeline intersect the shore to be built for the minimum cost?



- LET x BE THE DISTANCE FROM THE POINT ON SHORE DIRECTLY OPPOSITE THE DRILLING STATION TO WHERE THE PIPELINE HITS LAND.
- THE DISTANCE ON LAND IS $6-x$
- THE DISTANCE UNDERWATER IS $\sqrt{4+x^2}$

SO TOTAL COST OF PIPELINE IS

$$C(x) = 600\sqrt{4+x^2} + 200(6-x).$$

CRITICAL POINTS:

$$C'(x) = \frac{600x}{\sqrt{4+x^2}} - 200$$

$C'(x)=0$ IF $3x = \sqrt{4+x^2}$

OR $9x^2 = 4+x^2$

$8x^2 = 4$

$x = \pm \frac{1}{\sqrt{2}}$.

$x = \frac{1}{\sqrt{2}}$ IS RELEVANT CRIT. POINT.

MIN, SINCE $C'(x) < 0$ FOR $x < \frac{1}{\sqrt{2}}$

$C'(x) > 0$ FOR $x > \frac{1}{\sqrt{2}}$.

SHOULD HIT SHORE $\frac{1}{\sqrt{2}}$ MI FROM DRILLING STATION

Part 2: These will be graded **only** if you have passed part 1. **Name:** ____________

8 points	15. (a) For the curve given by $x^3 - 3y^4 = 4x^2y^3 - 6$, find dy/dx when $x = 1$ and $y = 1$.

BY IMPLICIT DIFF

$$3x^2 - 12y^3y' = 8xy^3 + 12x^2y^2y'$$

AT (1,1),

$$3 - 12y' = 8 + 12y'$$

$$-5 = 24y'$$

$$\frac{-5}{24} = y'.$$

5 points	(b) Use your answer to the previous part to estimate the y-value of a point on the curve with $x = 1.2$.

TANGENT LINE AT (1,1) IS

$$y = 1 - \frac{5}{24}(x-1)$$

AT $x = 1.2 = \frac{6}{5}$, WE HAVE

$$y = 1 - \frac{5}{24}\left(\frac{1}{5}\right) = 1 - \frac{1}{24} = \frac{23}{24}$$

Part 2: These will be graded **only** if you have passed part 1. **Name:** ______________

16. Consider the function $f(x) = 4x^5 + 5x^4 - 40x^3$.

$\boxed{\text{4 points}}$ (a) Find the x-values of all critical points of $f(x)$

$$f'(x) = 20x^4 + 20x^3 - 120x^2 = 20x^2(x^2+x-6)$$
$$= 20x^2(x+3)(x-2)$$

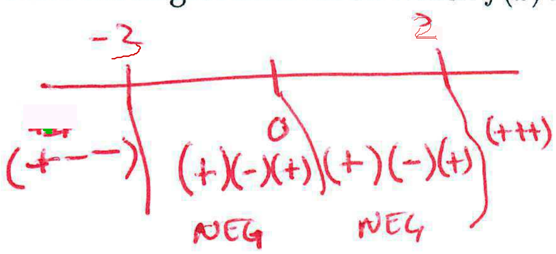
SO THERE ARE CRITICAL POINTS WHEN

$$20x^2(x+3)(x-2) = 0$$

THAT IS

$$x=0, \; x=-3, \text{ AND } x=2.$$

$\boxed{\text{4 points}}$ (b) State the largest interval on which $f(x)$ is decreasing.



IF $x < -3$, $f'(x) = (+)(-)(-) > 0$
SO INCREASING,

IF $-3 < x < 0$, $f'(x) < 0$

IF $0 < x < 2$, $f'(x) < 0$

IF $2 < x$, $f'(x) > 0$.

f IS DECREASING FOR $-3 < x < 2$

$\boxed{\text{4 points}}$ (c) Give the x-values at which the absolute maximum and absolute minimum values of f occur when $-1 \le x \le 3$.

JUST CHECK ENDPOINTS & CRITICAL POINTS.

$f(-1) = -4 + 5 + 40 = 41$ (IGNORE -3 SINCE NOT IN DOMAIN)

$f(0) = 0$

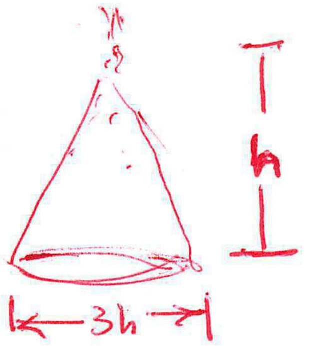
$f(3) = -112 < 0$

SO ABS MAX AT $x = -1$

ABS MIN AT $x = 3$.

Part 2: These will be graded **only** if you have passed part 1. **Name:** ______________

| 12 points | 17. Sand is falling from a chute at the rate of 144ft^3 per minute, and is forming a conical pile whose diameter is always three times its height. Find the rate at which the height of the pile is growing when the pile is 1 foot high. You might find it useful to know that the volume of a cone of radius r and height h is $\pi r^2 h/3$, its surface area is $\pi r\left(r + \sqrt{r^2+h^2}\right)$, or that ice cream cone was invented in 1896 by Italo Marchiony. Or maybe not.



LET h BE THE HEIGHT OF THE PILE, SO DIAMETER $= 3h$ (AND $r = 3h/2$)

SINCE THE SAND IS BEING ADDED AT 144 ft³/MIN, $\dfrac{dV}{dt} = 144$.

WE WANT dh/dt WHEN $h = 1$.

$$V = \pi r^2 h/3 = \pi\left(\frac{3h}{2}\right)^2\left(\frac{h}{3}\right) = \frac{3\pi}{4}h^3.$$

SO $$\frac{dV}{dt} = \frac{9\pi}{4}h^2\frac{dh}{dt}.$$

WHEN $h = 1$, WE HAVE

$$144 = \frac{9\pi}{4}\frac{dh}{dt}$$

SO $$\frac{2\cdot 288}{9\pi} = \frac{dh}{dt}$$

$$\frac{64}{\pi} = \frac{dh}{dt}$$

Name: ___________________

18. Let $R(x) = \dfrac{e^{2x} - 1}{\pi x}$.

8 points (a) Find a value k so that if we define $R(0) = k$, the resulting function is continuous. Fully justify your answer.

NOTE THAT $R(0) = \frac{1-1}{\pi \cdot 0} = \frac{0}{0}$, SO IT ISN'T A CONTINUOUS FUNCTION.

BUT USING L'HOPITALS (SINCE IT IS $0/0$), WE GET

$$\lim_{x \to 0} \frac{e^{2x}-1}{\pi x} = \lim_{x \to 0} \frac{2e^{2x}}{\pi} = \frac{2}{\pi}.$$

SO LET $R(0) = \frac{2}{\pi}$, AND

$$R(x) = \begin{cases} (e^{2x}-1)(\pi/x) & x \neq 0 \\ 2/\pi & x = 0 \end{cases}$$

IS CONTINUOUS.

5 points (b) Is the function in the previous part differentiable at all values of x? Fully justify your answer.

YES

FOR $x \neq 0$, $R(x)$ IS RATIONAL AND THE DENOMINATOR IS NON ZERO, SO IT IS DIFFERENTIABLE.

TO GET $R'(0)$, WE LOOK AT

$$R'(0) = \lim_{h \to 0} \frac{R(0+h) - R(0)}{h} = \lim_{h \to 0} \frac{\frac{e^{2h}-1}{\pi h} - \frac{2}{\pi}}{h}$$

$$= \lim_{h \to 0} \frac{e^{2h} - 1 - 2h}{\pi h^2} \overset{\text{L'HOPITAL}}{=} \lim_{h \to 0} \frac{2e^{2h} - 2}{2\pi h} \overset{\text{L'HOP}}{=} \lim_{h \to 0} \frac{4e^{2h}}{2\pi} = 2/\pi$$

SO YES