Math 125 Second Midterm

March 29, 2018

Name: _____ **ID:** _____ **Rec:** _____

Question:	1	2	3	4	5	6	7	Total
Points:	12	12	10	12	10	12	8	76
Score:								

There are 7 problems in this exam. Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, calculators, extra papers, and discussions with friends are not permitted.** If you brought a duck with you to the exam, you may consult it on any mathematical questions you may have. (Why a duck? Why a-no chicken?)

Points will be taken off for writing mathematically false statements, even if the rest of the problem is correct.

Use non-erasable pen (not red) if you want to be able to contest the grading of any problems. Questions with erasures will not be regraded.

Leave all answers in exact form (that is, **do not** approximate π, square roots, and so on.) In general, it is not necessary to simplify complicated expressions, although you **will** lose a point or two for not doing "trivial" simplifications like $\sqrt{4} = 2$, $15/5 = 3$, $\ln(1) = 0$, $\tan(\pi/4) = 1$ and so on. In many cases, some simplification is necessary at intermediate steps to arrive at the answer, but not always. If the problem tells you to simplify completely, you must do so.

You have 90 minutes to complete this exam.

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1. Compute the derivatives for each of the functions below.

4 points (a) $f(x) = 6x^5 - 5x^{-2} + \sqrt{2x} - \pi^4$.

4 points (b) $g(x) = x^6 e^{2x}$

4 points (c) $R(x) = \dfrac{x^3 + 8}{8 - 2x^3}$

2. Calculate the indicated derivatives.

4 points (a) $\dfrac{d}{dr}\left(\dfrac{2}{r}-\dfrac{r}{2}\right)$.

4 points (b) $L(x)=\ln\left(\dfrac{x^2-4}{x^2+4}\right)$. Find $L'(1)$.

4 points (c) $\dfrac{d}{dx}\ln(\sec 4x)$

3. Let $f(x) = xe^{-2x}$.

5 points (a) Calculate $f'(x)$.

5 points (b) For what values of x is $f(x)$ decreasing? If there are none, write "NONE"; otherwise, describe *all* such x. Give an exact answer (that is, do not approximate square roots, e, π, etc.) and justify your answer.

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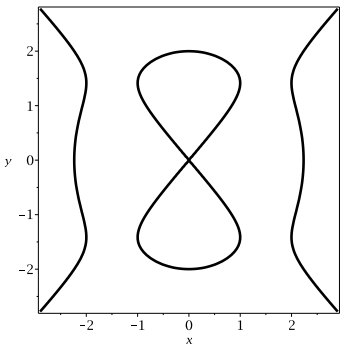
12 points 4. The set of points (x, y) which satisfy the relationship

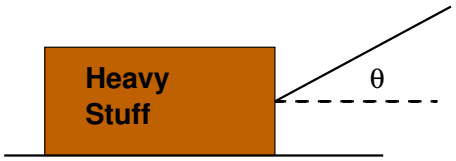
$$y^2(y^2 - 4) = x^2(x^2 - 5)$$

lie on what is known as a "devil's curve", shown at right.



Write the equation of the line tangent to the given devil's curve at the point $(-\sqrt{5}, 2)$.

5. When an object with weight W is dragged horizontally by a rope attached to the object which makes an angle θ with the horizontal, the force required has magnitude



$$F(\theta) = \frac{\mu W}{\mu \sin\theta + \cos\theta},$$

where the number μ is called the *coefficient of friction*.

5 points (a) If the object is a 60 lb box of rocks with a coefficient of friction μ of $1/2$, find $F'(\theta)$.

5 points (b) Suppose you are pulling the box of rocks and the rope makes an angle of $\pi/6$ with the horizontal. Does it make sense to lower the rope (that is, decrease θ)? Explain why or why not (no points for a correct answer without a valid explanation).

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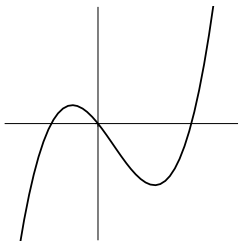
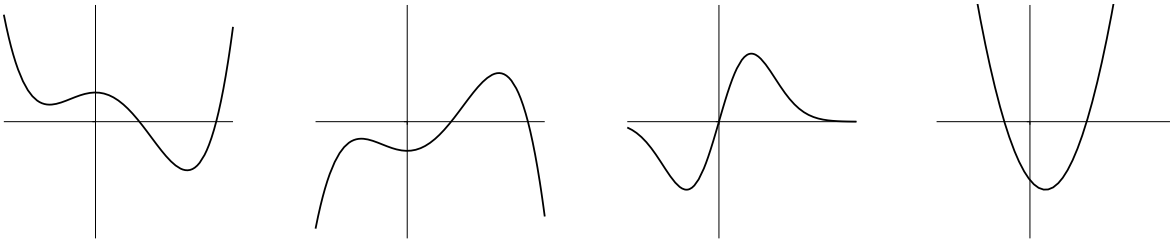
6. At right is the graph of **the derivative** $f'(x)$ of a function $f(x)$. Use it to answer each of the following questions.

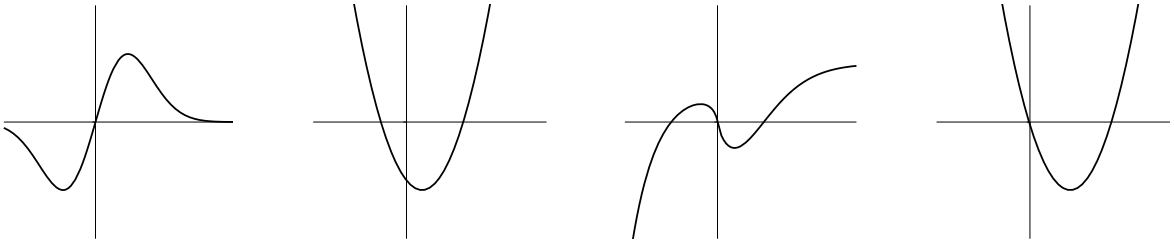


4 points (a) Is $f(x)$ concave up, concave down, or neither at $x = 0$? Fully justify your answer.

4 points (b) Which of the following best represents the graph of $f(x)$? (circle your answer).



4 points (c) Which of the following best represents the graph of $f''(x)$? (circle your answer).



8 points 7. A function $H(x)$ measures the total "Hapyness" of a cube (yes, it is spelled that way—cubes don't spel so gud), depending on its width x. The Hapyness per unit volume is given by $Q(x) = \dfrac{H(x)}{x^3}$. If $H(10) = 2$ and $H'(10) = -6$, compute the rate of change of Hapyness per unit volume when $x = 10$, that is, find $Q'(10)$.