$\qquad$ Rec:

| Question: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Total |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Points: | 15 | 6 | 10 | 6 | 8 | 16 | 8 | 8 | 77 |
| Score: |  |  |  |  |  |  |  |  |  |

There are 8 problems in this exam. Make sure that you have them all.
Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. Books, calculators, extra papers, and discussions with friends are not permitted. If you have a time machine, feel free to go ahead a few days and take a look at the solutions. However, you must then loan the machine to me so that I can go back and change the questions.

Leave all answers in exact form (that is, do not approximate $\pi$, square roots, and so on.)

You have 90 minutes to complete this exam.
$\qquad$

1. Compute each of the following limits. If the limit is not a finite number, please distinguish between $+\infty,-\infty$, and a limit which does not exist (DNE). Justify your answer, at least a little bit.
(a) 3 points $\lim _{x \rightarrow 0} \frac{\sin x}{\tan x}$
(b) 3 points $\lim _{x \rightarrow+\infty} \frac{5 x^{2}-4 x-1}{x^{2}-1}$
(c) 3 points $\lim _{x \rightarrow+\infty} \sqrt{4 x^{2}+x}-2 x$
(d) 3 points $\lim _{x \rightarrow 0^{-}} \frac{1}{x^{5}}$
(e) 3 points $\lim _{x \rightarrow 0} \frac{(2+x)^{2}-4}{x}$
$\qquad$
2. 6 points Let

$$
f(x)= \begin{cases}3 x^{2} & \text { if } x<-1 \\ 3 \tan \left(\frac{\pi}{4} x\right) & \text { if }-1 \leq x \leq 1 \\ 3 x^{3} & \text { if } x>1\end{cases}
$$

For which values of $x$ is $f(x)$ continuous? Justify your answer.
3. Let $f(x)=2 x^{3}-4 x+4$.
(a) 5 points Find $f^{\prime}(1)$.
(b) 5 points Write the equation of the line tangent to $f(x)$ at the point $P=(1,2)$.
$\qquad$
4. 6 points Write a limit that represents the slope of the graph

$$
y= \begin{cases}8+x \ln |x| & x \neq 0 \\ 8 & x=0\end{cases}
$$

at $x=0$. You do not need to evaluate the limit.
5. At right is the graph of the derivative $f^{\prime}$ of a function.
(a) 4 points List all values of $x$ with $-3 \leq x \leq 4$ where $f(x)$ has a local maximum.

(b) 4 points At $x=-1$, is $f(x)$ concave up, concave down, or neither?
$\qquad$
6. 16 points For each of the 4 functions graphed in the left column, find the corresponding derivative function among any of the 8 choices on the right (not just on the same row) and put its letter in the corresponding box.

A:


B:


C:

D:


E:

F:


G:

H:

$\qquad$
7. Let $f(x)=\frac{4-x^{2}}{(3+x)^{2}}$.
(a) 4 points Identify the horizontal asymptotes of $f(x)$. If there are none, write "NONE".
(b) 4 points Identify the vertical asymptotes of $f(x)$. If there are none, write "NONE".
8. 8 points An exponential function of the form $y=C a^{x}$ passes through the points $(1,6)$ and $(3,24)$. Find $C$ and $a$.

