MATH 125 First Midterm

February 19, 2018

Name: _____ **ID:** _____ **Rec:** ____

Question:	1	2	3	4	5	6	7	8	Total
Points:	15	15	15	14	10	20	10	10	109
Score:									

There are 8 problems in this exam. Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, calculators, extra papers, and discussions with friends are not permitted.** If you used online materials to study, beware that you might have been the victim of Russian or North Korean bots spreading `#FakeMath`. If so, you might want to report it to Robert Mueller.

Points will be taken off for writing mathematically false statements, even if the rest of the problem is correct.

Use non-erasable pen (not red) if you want to be able to contest the grading of any problems. Questions with erasures will not be regraded.

Leave all answers in exact form (that is, **do not** approximate π, square roots, and so on.) In general, it is not necessary to simplify complicated expressions, although you **will** lose a point or two for not doing "trivial" simplifications like $\sqrt{4} = 2$, $15/5 = 3$, $\ln(1) = 0$, $\tan(\pi/4) = 1$ and so on. In many cases, some simplification is necessary at intermediate steps to arrive at the answer, but not always. If the problem tells you to simplify completely, you must do so.

You have 90 minutes to complete this exam.

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1. A tank holding 1200 gallons of maple syrup can be drained completely in three hours by opening a valve at its bottom. The amount of syrup in the tank at time t (where t is in hours, with $0 \leq t \leq 3$) is given by

$$V(t) = 1200\left(1 - \frac{t}{3}\right)^2 = \frac{1200}{9}(3-t)^2.$$

5 points (a) Calculate the average rate (in gallons per hour) at which the syrup drains between $t = 1$ hour and $t = 2$ hours.

5 points (b) Write a formula that represents the average rate (in gallons per hour) at which the syrup drains between 2 hours some time t hours (with $0 \leq t \leq 3$, $t \neq 2$).

5 points (c) Calculate the instantaneous rate (in gallons per hour) at which the syrup is draining out at time $t = 2$ hours (depending on how you write it, this could be negative).

2. Compute each of the following limits. If the limit is not a finite number, please distinguish between $+\infty$, $-\infty$, and a limit which does not exist (DNE). Justify your answer, at least a little bit.

5 points (a) $\lim_{x \to \pi/4} x^2 \tan x$

5 points (b) $\lim_{x \to 4} \dfrac{x^2 - 16}{5x^2 - 20x}$

5 points (c) $\lim_{x \to 3^+} \dfrac{x^3 - 4x^2 + 1}{(x-5)(x-3)}$

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3. More of the same: compute each of the following limits. If the limit is not a finite number, please distinguish between $+\infty$, $-\infty$, and a limit which does not exist (DNE). Justify your answer, at least a little bit.

5 points (a) $\lim_{x\to-\infty} \dfrac{9x^5-1}{3x^5-27x^3+10}$

5 points (b) $\lim_{x\to 4} \dfrac{x^2-16}{|x-4|}$

5 points (c) $\lim_{x\to 4} \dfrac{\sqrt{x}-2}{x-4}$

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4. For $0 < x < 2$, let $h(x) = \dfrac{4}{x} + \dfrac{4}{x-2} + \cos(\pi x)$.

2 points (a) What is $h(1)$? Simplify as much as possible, or write "Not Defined" if it is not defined.

5 points (b) Calculate $\lim_{x \to 0^+} h(x)$ and $\lim_{x \to 2^-} h(x)$. If either limit is not a number, distinguish between $+\infty$, $-\infty$, and DNE (does not exist).

7 points (c) Is there a value of x between 0 and 2 so that $h(x) = 0$?
If so, is the value of x bigger than, equal to, or less than 1? If not, why not?
How do you know? (Fully justify your answer.)

10 points 5. Explain in words what $\lim_{x \to 3} f(x) = +\infty$ means.

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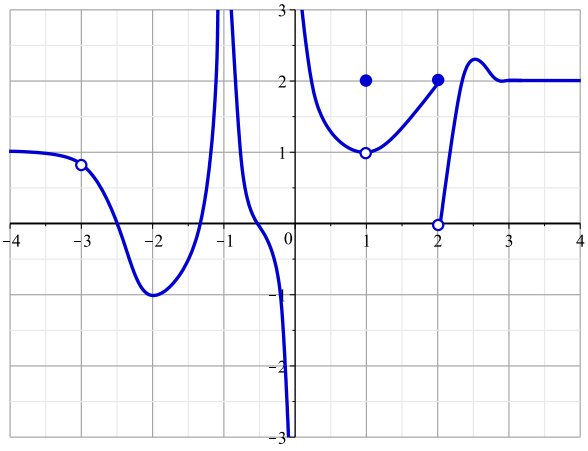
20 points

6. The graph of a function $f(x)$ is shown at right, for $-4 < x < 4$. Answer the questions below, assuming that for values not shown on the graph, the function continues in the same way (i.e., essentially straight).
When calculating limits, distinguish between $+\infty$, $-\infty$, and "does not exist".



(a) What is $\lim_{x\to+\infty} f(x)$, if it exists?

(b) What is $\lim_{x\to 2^+} f(x)$, if it exists?

(c) What is $\lim_{x\to 2} \Big[(x-2)f(x)\Big]$, if it exists?

(d) List all points x between -4 and 4 where $f(x)$ is defined, but $f(x)$ is not continuous.

10 points 7. What value of k is necessary so that the function

$$f(x) = \begin{cases} kx + 5 + \ln 2 & x < 1 \\ 2kx^2 + x & x \geq 1 \end{cases}$$

is continuous for all positive values of x? If there is no such value, write "NONE". Justify your answer fully.

10 points 8. Write a limit that represents the slope of the line tangent to the graph of the function

$$f(x) = \begin{cases} (1 - x)\tan(\frac{\pi}{2}x) & x \neq 1 \\ \frac{2}{\pi} & x = 1 \end{cases}$$

at $x = 1$. You **do not need to evaluate the limit.**